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**VEHICLE-TO-VEHICLE COMMUNICATION  
FOR COLLISION AVOIDANCE AND  
IMPROVED TRAFFIC FLOW**

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**INNOVATIONS DESERVING EXPLORATORY ANALYSIS (IDEA) PROGRAMS  
MANAGED BY THE TRANSPORTATION RESEARCH BOARD (TRB)**

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This investigation was completed as part of the ITS-IDEA Program which is one of three IDEA programs managed by the Transportation Research Board (TRB) to foster innovations in surface transportation. It focuses on products and result for the development and deployment of intelligent transportation systems (ITS), in support of the U.S. Department of Transportation's national ITS program plan. The other two IDEA programs areas are Transit-IDEA, which focuses on products and results for transit practice in support of the Transit Cooperative Research Program (TCRP), and NCHRP-IDEA, which focuses on products and results for highway construction, operation, and maintenance in support of the National Cooperative Highway Research Program (NCHRP). The three IDEA program areas are integrated to achieve the development and testing of nontraditional and innovative concepts, methods and technologies, including conversion technologies from the defense, aerospace, computer, and communication sectors that are new to highway, transit, intelligent, and inter-modal surface transportation systems.

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## EXECUTIVE SUMMARY

### PROBLEM STATEMENT

Our objective is to determine the performance requirements of a vehicle-to-vehicle communication and on-board processing system which would provide prediction and driver warning of potential inter-vehicular collisions, improving collision avoidance.

The basic premise of this work is that congestion, delay, accidents, and other associated problems are often a result of the typical driver's inability to correctly assess the current and impending driving situations. The driver has incomplete information about the speed, acceleration, position, etc. of other vehicles, especially vehicles occluded by intervening vehicles. Thus, drivers are forced to make basic operating decisions such as when to brake, based on incomplete information.

Previous research on vehicle-to-vehicle, vehicle-to-roadside, and roadside-to-vehicle communication has focused either on wide-area information such as link times for the purposes of route guidance, or on very short-range and high-frequency communication for the purposes of vehicle control. The approach taken in this project is unique in that the warning system developed allows vehicles to operate independently and without the control problems associated with platooning, but with the ability to utilize data from several vehicles ahead, unlike two-vehicle interaction systems. In this project, we are studying an intermediate level of communication between nearby vehicles, which is short of platooning and avoids some of the problems of platoon-type systems, but goes beyond two-vehicle interaction systems such as Intelligent Cruise Control (ICC) by considering data from multiple vehicles in the local area.

The impact of the developed advisory system is potentially large. Some 20% of all accidents are of the rear-end collision type (1, 2, 3). Such accidents result in substantial loss of life and substantial cost in the form of delay and property loss. Researchers have stated that 60% of all rear-end collisions could be avoided if the driver were given an additional one-half second of warning prior to an incident (4). Our work addresses this issue by providing advanced collision warning.

### RESEARCH APPROACH

Each equipped vehicle in the system is assumed to have a sensor for sensing the vehicle immediately ahead, communications devices, and an on-board computer. As an equipped vehicle travels along the roadway, it continuously broadcasts data to other nearby equipped vehicles. The data includes speed, position, and acceleration. The communication system would most likely utilize spread-spectrum techniques, largely due to their ability to provide multiple access to the same frequency, for example via code division multiple access (CDMA), and their low interference to other communications systems. As the information is obtained from other vehicles in the local area, the software in the on-board computer builds and updates its model of the local environment. This model is potentially more complete than the driver's model because of the driver's limited information and limited ability to accurately interpret the information that is available. The system utilizes the computed model to advise the driver of a recommended action.

Since each equipped vehicle's computer will have knowledge of information such as the position, velocity, and acceleration of nearby equipped vehicles, the amount of deceleration, if any, required to avoid a collision with the vehicle directly ahead can be determined. This would aid in avoiding common rear-end and multiple-car collisions as well as those caused by severe weather conditions such as fog, or inoperable tail lights on the vehicle ahead. To this end, we have developed an algorithm which yields the deceleration required by a vehicle,  $V$ , to avoid a rear-end collision with the vehicle ahead. This algorithm takes into consideration the response of the vehicle immediately ahead to the vehicle ahead of it, the latter's response to the vehicle ahead of it, and so on, within  $v$ 's communication range. The algorithm utilizes more information, and thus provides a more accurate evaluation of the required deceleration, than previous distance-warning systems.

Initially, we assumed that all the vehicles in the system were equipped (i.e. 100% penetration of the system). Under this assumption, we established operating parameters for the system, and determined the system's potential benefit. We then extended the study to address issues of system deployment (i.e. less than 100% penetration) by studying the impact of various levels of system penetration.

### CONCLUSIONS

To determine the potential benefit of this algorithm, we ran a series of experiments using a quasi-Monte Carlo approach, and data collected by the FHWA. The data used was comprised of two sets of data, each representing a day of

traffic data collected on Interstate-40 in New Mexico. To study the impact of the system, we generated an incident by assuming a more or less severe deceleration for a given vehicle in a chain of vehicles. We studied the degree to which the system is effective in 1) reducing the number of collisions, 2) reducing the impact of collisions which do occur, and 3) reducing the required braking force when a collision is averted. We chose these three factors for the following reasons. First, reducing the number of collisions has obvious benefits in terms of reducing injuries and vehicle damage, as well as reducing congestion and delay which accidents produce. Second, reducing the impact of collisions which do occur also translates to reductions in injuries and vehicle damage due to the reduction in the force of the collision. Finally, reducing the braking force when a collision is averted adds to driver comfort, and it also helps to reduce the amount of disturbance introduced in the vehicles following the decelerating vehicle.

Assuming that all the vehicles in the system are equipped (i.e. 100% penetration of the system), we can summarize the conclusions drawn from these experiments as follows:

- significant gain is obtained by processing data from up to 6 or 7 vehicles ahead, but very little or no benefit is obtained by utilizing data from more than 6 or 7 vehicles ahead.
- significant gain is obtained by processing data from vehicles within a 600 ft. or 700 ft. communication range, but very little or no benefit is obtained for communication ranges above 600 ft. or 700 ft.
- the number of vehicles ahead from which to process data, and the appropriate communication range are relatively unaffected by the severity of the lead vehicle's deceleration.
- variability in the driver's reaction time from the expected value has relatively little effect on the number of collisions, and the deceleration rate required to avert collisions.
- variability in the driver's deceleration rate from the required rate calculated by the algorithm has relatively little effect on the number of collisions, and the deceleration rate required to avert collisions.

We can summarize the conclusions drawn from the deployment experiments (less than 100% penetration) as follows:

- the system is effective in reducing the number of collisions even during deployment.
- broadcasting data from non-transmitting vehicles which are sensed significantly increases the effective system penetration, providing earlier system benefits.
- the system is dramatically effective in reducing collisions in the case of limited sight visibility (e.g. fog)

The above results allow us to place reasonable limitations on the number of vehicles from which to process data, and on the communication range required by the system. In addition, they indicate that driver variability has relatively little effect on the potential benefit of the system. Thus, the results of the evaluation indicate that significant benefits in terms of reduction in the number of collisions, reduction in the impact of collisions which do occur, and reduction in the required braking force when a collision is averted are obtained by a system which is capable of processing data from 6 or 7 vehicles ahead, as opposed to processing data from 3 or fewer vehicles ahead which is more typical of an unassisted driver. These benefits are realized even during deployment of the system.

# COLLISION WARNING USING VEHICLE-TO-VEHICLE COMMUNICATION

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## 1.0 PROBLEM STATEMENT

Our objective is to determine the performance requirements of a vehicle-to-vehicle communication and on-board processing system which would provide prediction and driver warning of potential inter-vehicular collisions, improving collision avoidance.

The basic premise of this work is that congestion, delay, accidents, and other associated problems are often a result of the typical driver's inability to correctly assess the current and impending driving situations. The driver has incomplete information about the speed, acceleration, position, etc. of other vehicles, especially vehicles occluded by intervening vehicles. Thus, drivers are forced to make basic operating decisions such as when to brake, based on incomplete information.

## 1.1 BACKGROUND

In the area of collision warning/avoidance, previous researchers, including those participating in the DRIVE, PROMETHEUS, and PATH projects, have focused either on two-vehicle interaction, that is on the interaction of the driver's vehicle with the vehicle immediately ahead without regard for other vehicles (5, 6, 7, 8, 9, 10, 11, 12, 13), or on vehicle entrainment or platooning where each vehicle has data from the vehicle ahead as well as the lead vehicle in the platoon (7, 13, 14, 15, 16, 17, 18, 19).

### 1.1.1 Two-Vehicle Interaction Systems

Many proposed collision warning systems make use of a sensor which returns the distance to the vehicle ahead. These systems can only obtain the distance to the vehicle ahead and the relative velocity between the vehicles (by calculating the change in distance over time). They typically use the sensed data to determine a "safe distance." Drivers whose vehicles are at a distance less than the safe distance receive a warning either visually, audibly, or via sensory feedback, such as increasing the force required to operate the accelerator or by pulsing the accelerator pedal (11).

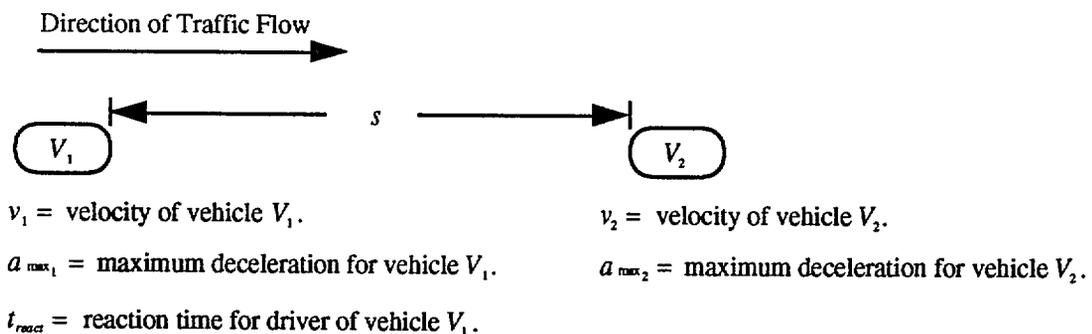


FIGURE 1 Two-vehicle interaction in a single lane.

If we consider the two-vehicle situation depicted in Figure 1, one calculation of the safe distance,  $s_{safe}$ , between vehicles is given by (11),

$$s_{safe} = \frac{v_1^2}{2 \cdot a_{max_1}} + t_{react} \cdot v_1.$$

This expression yields an absolute safe distance, but provides an overly conservative estimate of  $s_{safe}$  because it assumes that the lead vehicle is not moving. A better estimate of the safe distance is given by (5, 9, 10, 13),

$$s_{safe} = \frac{v_1^2}{2 \cdot a_{max_1}} - \frac{v_2^2}{2 \cdot a_{max_2}} + t_{react} \cdot v_1.$$

This expression provides a better estimate of the safe distance, but inherent in this equation is the worst case assumption that the lead driver decelerates as rapidly as possible, that is with deceleration  $a_{max_2}$ . This conservative estimate is based on the fact that no information is available from the vehicles further ahead. If that information were available, we could use it to estimate the response of vehicle  $V_2$  based on the vehicles ahead of it. This is what we have done in the system developed under this project.

### 1.1.2 Platoon-Type Systems

In a platoon-type system depicted in Figure 2, each vehicle has data from the vehicle immediately ahead as well as the lead vehicle in the platoon (7, 13, 14, 15, 16, 17, 18, 19). The platoons are assumed to be separated by a distance sufficiently large as to allow them to be treated independently. A problem with these systems is that in order to maintain the very short headways desired between vehicles (~ 1 m.), the driver must relinquish control of the vehicle. This increases problems with driver acceptance and reliability, and adds many potential liability problems.

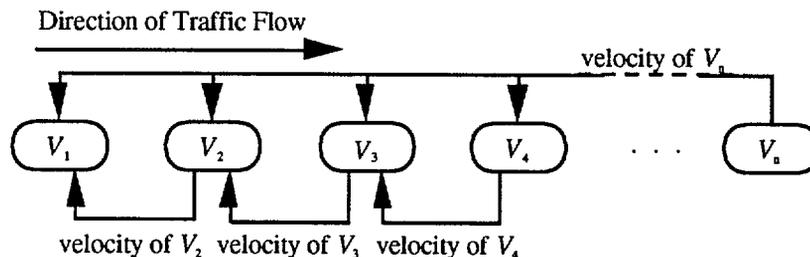


FIGURE 2 A platoon of vehicles.

## 1.2 INNOVATION

The approach taken in this project is unique in that the warning system developed allows vehicles to operate independently and without the control problems associated with platooning, but with the ability to utilize data from several vehicles ahead, unlike two-vehicle interaction systems. In this project, we are studying an intermediate level of communication between nearby vehicles, which is short of platooning and avoids some of the problems of platoon-type systems, but goes beyond two-vehicle interaction systems such as Intelligent Cruise Control (ICC) by considering data from multiple vehicles in the local area.

The impact of the developed advisory system is potentially large. Some 20% of all accidents are of the rear-end collision type (1, 2, 3). Such accidents result in substantial loss of life and substantial cost in the form of delay and property loss. Researchers have stated that 60% of all rear-end collisions could be avoided if the driver were given an additional one-half second of warning prior to an incident (4). Our work addresses this issue by providing advanced collision warning.

## 2.0 RESEARCH APPROACH

Each equipped vehicle in the system is assumed to have a sensor for sensing the vehicle immediately ahead, communications devices, and an on-board computer. As an equipped vehicle travels along the roadway, it continuously broadcasts data to other nearby equipped vehicles. The data includes speed, position, and acceleration. The

communication system would most likely utilize spread-spectrum techniques, largely due to their ability to provide multiple access to the same frequency, for example via code division multiple access (CDMA), and their low interference to other communications systems. As the information is obtained from other vehicles in the local area, the software in the on-board computer builds and updates its model of the local environment. This model is potentially more complete than the driver's model because of the driver's limited information and limited ability to accurately interpret the information that is available. The system utilizes the computed model to advise the driver of a recommended action.

Since each equipped vehicle's computer will have knowledge of information such as the position, velocity, and acceleration of nearby equipped vehicles, the amount of deceleration, if any, required to avoid a collision with the vehicle directly ahead can be determined. This would aid in avoiding common rear-end and multiple-car collisions as well as those caused by severe weather conditions such as fog, or inoperable tail lights on the vehicle ahead. To this end, we have developed an algorithm which yields the deceleration required by a vehicle,  $V_i$ , to avoid a rear-end collision with the vehicle ahead. This algorithm takes into consideration the response of the vehicle immediately ahead to the vehicle ahead of it, the latter's response to the vehicle ahead of it, and so on, within  $V_i$ 's communication range. The algorithm utilizes more information, and thus provides a more accurate evaluation of the required deceleration, than previous distance-warning systems.

Initially, we assumed that all the vehicles in the system were equipped (i.e. 100% penetration of the system). Under this assumption, we established operating parameters for the system, and determined the system's potential benefit. We then extended the study to address issues of system deployment (i.e. less than 100% penetration) by studying the impact of various levels of system penetration.

In the following sections, we discuss the development of the algorithm, the method of evaluation used to determine certain parameters used in the algorithm, and the potential impact of the warning device in operation both in the case of 100% system penetration and during deployment.

## 2.1 ALGORITHM DEVELOPMENT

### 2.1.1 Introduction

The purpose of the following analysis, starting with the two-vehicle case, is to determine what action a vehicle must take in order to avoid a collision with the vehicle immediately ahead. The analysis will take into account the fact that the lead vehicle may itself be taking action to avoid a collision with the vehicle ahead of it, and so on. The set of equations developed will yield a (constant) deceleration value that the trailing vehicle must employ to avoid a collision with the vehicle ahead. These equations will take into consideration the human reaction time of the drivers involved.

The scenario of interest is shown in Figure 3. Here we have two vehicles traveling in a single lane in the same direction. At a given moment, we can characterize a vehicle,  $V_i$ , by its distance traveled along the roadway,  $s_i(t)$ , its velocity,  $v_i(t)$ , and its acceleration,  $a_i(t)$ , which is positive if the vehicle is speeding up, and negative if the vehicle is slowing down.

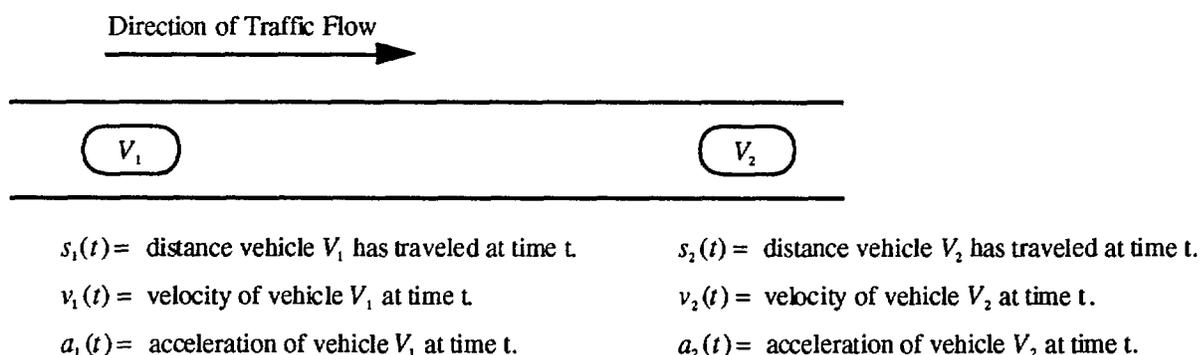


FIGURE 3 Two vehicles traveling in a single lane.

If we wish to avoid a collision between vehicles  $V_1$  and  $V_2$ , we want the distance traveled by  $V_1$  to always be less than or equal to the distance traveled by  $V_2$ . That is, we require that  $s_1(t) \leq s_2(t)$ , for all  $t$ . (At this time, we will disregard the lengths of the vehicles in an effort to simplify the explanation of the problem, but we will consider the lengths of the vehicles below in the formal development of the equations.)

We can graphically determine if a collision occurs by plotting the distances traveled by each vehicle over time. In Figure 4, we plot the distance traveled versus time for a single vehicle. In this simple case, the vehicle has zero acceleration, and thus the curve is linear. The slope of this line is the velocity of the vehicle.

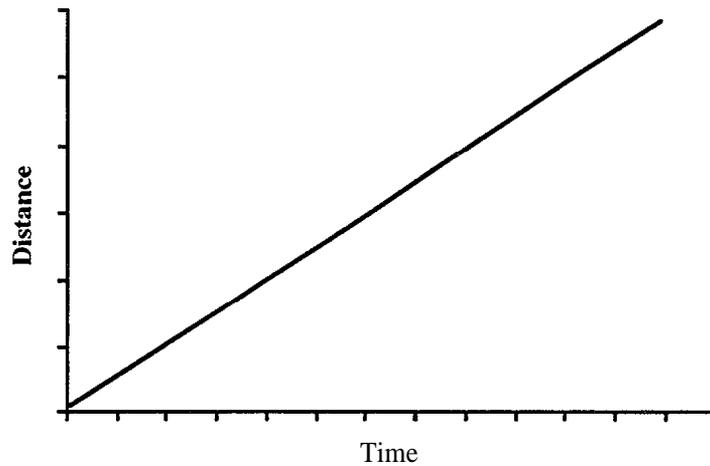


FIGURE 4 Single vehicle with zero acceleration.

If we plot the position of two vehicles versus time on the same chart, we can determine if impacts occur by looking for intersections of the curves. In Figure 5, we plot two vehicles each with zero acceleration. In this example, the trailing vehicle has a larger velocity than the lead vehicle, and thus the lines intersect at the point of impact for the vehicles. Here the initial distance between the vehicles can be seen as the difference in vertical axis intercepts for the two lines.

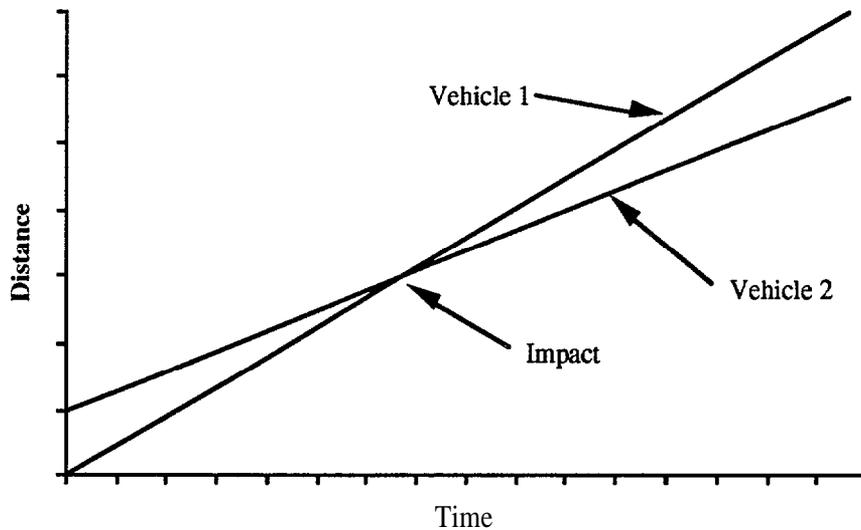


FIGURE 5 Two vehicles each with zero acceleration.

In both Figures 4 and 5, we have considered vehicles with zero acceleration. In Figure 6, we consider a single vehicle undergoing a constant deceleration. In this case, the vehicle travels at a constant velocity (zero acceleration) until time  $b$  when the deceleration begins. This deceleration continues until time  $e$  when the deceleration ends, and the vehicle continues at a constant, but lower than initial, velocity. This is again somewhat of a simplification, since the vehicle may in fact be accelerating or decelerating at a different rate between time 0 and  $b$ , and after time  $e$ , but to simplify the figures, we will depict this acceleration as zero. The equations to be developed will allow for non-zero acceleration between time 0 and  $b$ , and after time  $e$ .

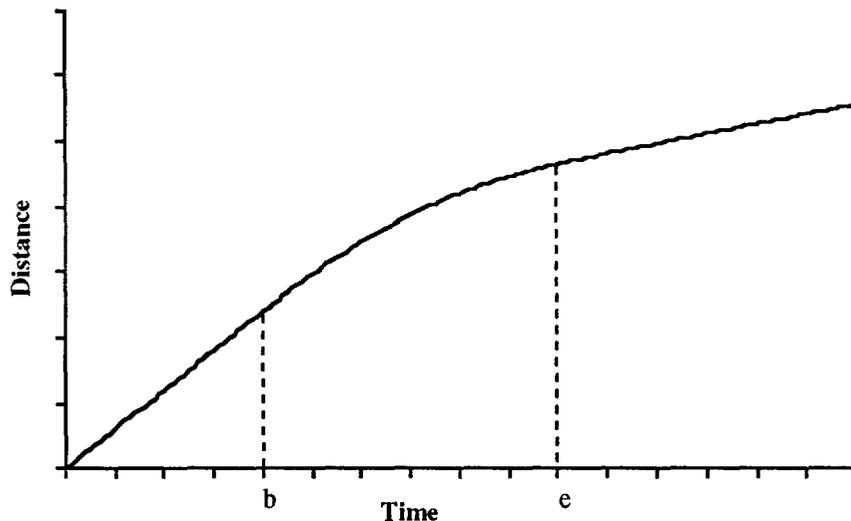


FIGURE 6 Single vehicle during deceleration.

When we consider two vehicles, as in Figure 3, our goal is to determine the action that the trailing vehicle must take to avoid a collision with the vehicle ahead, which may be decelerating (or accelerating) in response to the situation ahead of it. That is, we need to determine what, if any, (constant) deceleration is required by the trailing vehicle to avoid intersecting the distance-versus-time curve of the vehicle ahead.

### 2.1.2 Required Deceleration

In calculating the required deceleration, we assume that we have knowledge of the behavior of the vehicle immediately ahead. That is, we know that vehicle's expected distance-versus-time curve for the near term. This behavior will be determined by considering the response of the vehicle ahead to the vehicle ahead of it, and so on, and will be addressed in Section 2.1.3. In determining the required deceleration, we also need to consider the human reaction time of the driver of the trailing vehicle.

When we have a non-zero reaction time for the trailing vehicle, there are two general scenarios which can occur. The first, and worst case, is that the warning comes too late given the driver's reaction time. That is, the driver impacts the vehicle ahead before the deceleration in response to the warning begins. In the other case, the driver is able to react prior to impact, and assuming sufficient braking power, the impact can be avoided. In reality, the required deceleration rate may be beyond the physical limit of the vehicle, in which case an impact will still occur. We will examine these two cases in the following two sections.

#### 2.1.2.1 Collision Prior to Driver Reaction

In this case, we assume a collision between the vehicles occurs before the driver of the trailing vehicle can react to the warning. It is hoped that instances of this type will be few and far between, but if an impact is inevitable, we would like to reduce its severity by lessening the difference in velocities at the time of impact.

Figure 7a-c depicts the three possible crash scenarios for this case. (In each of these, we have simplified the figure by showing the trailing vehicle traveling at constant velocity prior to impact. The equations to follow make no such assumption.) In Figure 7a, the trailing vehicle,  $V_1$ , impacts the lead vehicle,  $V_2$ , prior to any deceleration by  $V_2$  in response to the vehicle ahead of it. In Figure 7b,  $V_1$  impacts  $V_2$  while it is decelerating to avoid the vehicle ahead of it, and in Figure 7c,  $V_1$  impacts  $V_2$  after  $V_2$ 's response to the vehicle ahead.

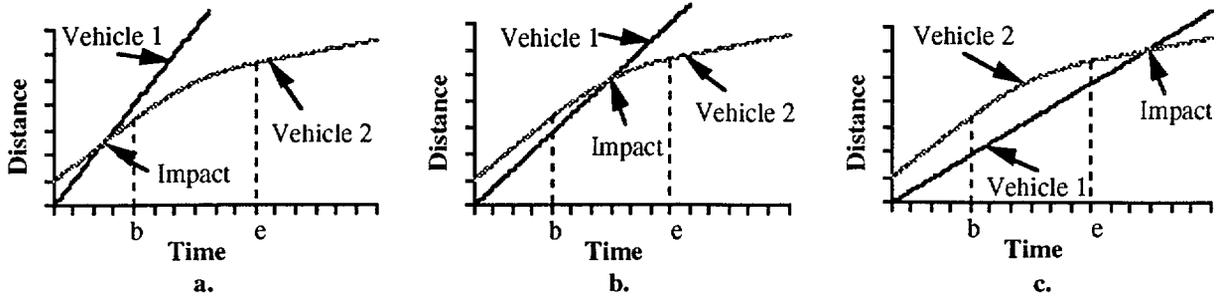


FIGURE 7a-c Three possible crash scenarios for a collision prior to trailing-driver reaction.

If we assume, as in Figure 6, that vehicle  $V_2$  sustains a constant acceleration rate<sup>\*</sup>,  $a_2(0)$ , (not necessarily zero) between time 0 and  $b_2$  a constant acceleration rate,  $a_2(b_2)$ , between time  $b_2$  and  $e_2$ , and a constant acceleration rate,  $a_2(e_2)$ , after time  $e_2$ <sup>\*\*</sup>, we can describe  $V_2$ 's position and velocity at any time  $t$  by the following sets of equations.

$$v_2(t) = \begin{cases} v_2(0) + a_2(0) \cdot t, & t \leq b_2 \\ v_2(b_2) + a_2(b_2) \cdot (t - b_2), & b_2 < t \leq e_2 \\ v_2(e_2) + a_2(e_2) \cdot (t - e_2), & t > e_2 \end{cases}$$

$$s_2(t) = \begin{cases} s_2(0) + v_2(0) \cdot t + \frac{1}{2} \cdot a_2(0) \cdot t^2, & t \leq b_2 \\ s_2(b_2) + v_2(b_2) \cdot (t - b_2) + \frac{1}{2} \cdot a_2(b_2) \cdot (t - b_2)^2, & b_2 < t \leq e_2 \\ s_2(e_2) + v_2(e_2) \cdot (t - e_2) + \frac{1}{2} \cdot a_2(e_2) \cdot (t - e_2)^2, & t > e_2 \end{cases}$$

Or, in general,

$$v_2(t) = v_2(\tau_2) + a_2(\tau_2) \cdot (t - \tau_2) \quad (1)$$

$$s_2(t) = s_2(\tau_2) + v_2(\tau_2) \cdot (t - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (t - \tau_2)^2 \quad (2)$$

where,

$$\tau_2 = \begin{cases} 0, & t \leq b_2 \\ b_2, & b_2 < t \leq e_2 \\ e_2, & t > e_2 \end{cases}$$

Similarly, we can establish equations for the velocity and position of  $V_1$  as a function of  $t$ . In this case, since the impact occurs before the driver of  $V_1$  responds to the warning, we assume that the acceleration of  $V_1$  is a constant,  $a_1(0)$ , prior to impact. Thus, for vehicle  $V_1$ , we have,

$$v_1(t) = v_1(0) + a_1(0) \cdot t \quad (3)$$

$$s_1(t) = s_1(0) + v_1(0) \cdot t + \frac{1}{2} \cdot a_1(0) \cdot t^2. \quad (4)$$

Since the impact occurs prior to the driver reaction, we cannot avert the impact, but we can determine the time of

\* This is an assumption which will be a good approximation to reality for a short period of time. If we recalculate everything very often, such as every 10 ms., it will generally be a very good approximation.

\*\* These assumptions will be made for all vehicles considered in this paper.

the impact and the difference in velocities at the time of impact. Impact occurs at time  $t$ , when  $s_2(t) - s_1(t) = L_2$ , where  $L_2$  is the length of vehicle  $V_2$  (assuming the position is measured from the front of each vehicle).  $L_2$  could be increased to include a desired gap to be maintained between vehicles, and/or any expected error in position.

Substituting Equations 2 and 4 into  $s_2(t) - s_1(t) = L_2$ , we have,

$$s_2(\tau_2) + v_2(\tau_2) \cdot (t - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (t - \tau_2)^2 - [s_1(0) + v_1(0) \cdot t + \frac{1}{2} \cdot a_1(0) \cdot t^2] = L_2.$$

Or,

$$\frac{1}{2} \cdot [a_2(\tau_2) - a_1(0)] \cdot t^2 + [v_2(\tau_2) - a_2(\tau_2) \cdot \tau_2 - v_1(0)] \cdot t + [s_2(\tau_2) - v_2(\tau_2) \cdot \tau_2 + \frac{1}{2} \cdot a_2(\tau_2) \cdot \tau_2^2 - s_1(0) - L_2] = 0.$$

This is a quadratic equation, and can be solved for  $t$ , using the quadratic formula. (This may produce two results. We choose the smallest value for  $t$ , that satisfies the conditions described below.) From the resulting expression for  $t$ , we can determine if an impact will occur prior to the time the driver reacts. We first assume that the impact will occur prior to  $b_2$ . That implies that  $\tau_2 = 0$  in the expression for  $t$ . If the resulting value for  $t$  is in fact less than  $b_2$ , then the collision occurs at the calculated value of  $t$ , and the difference in velocities at impact can be determined by substituting the value of  $t$  for  $t$  in Equations 1 and 3. If the solution value for  $t$  is greater than  $b_2$ , then the collision, if any, occurs at some time after  $b_2$ . We then assume that the impact occurs between time  $b_2$  and  $e_2$ , letting  $\tau_2 = b_2$ . We again solve for  $t$ , and check if it falls between  $b_2$  and  $e_2$ . If it does, we can determine the difference in velocities at impact as above. If not, we assume the impact occurs after  $e_2$ , let  $\tau_2 = e_2$ , and repeat the procedure. If it turns out that the impact occurs after the reaction time,  $r_1$ , of the driver of vehicle,  $V_1$ , then the driver has time to react to the warning. The level of warning to give, in that case, is calculated in the following section.

### 2.1.2.2 Deceleration Required after the Driver Reacts

In the second case, the driver of the trailing vehicle reacts to the warning prior to an impact with the driver immediately ahead. Assuming that the vehicle has sufficient braking ability, the impact can be avoided. In this section, we will determine the amount of deceleration required to avoid impact with the vehicle ahead. Once we know the amount of deceleration that is required, we can compare it to the deceleration limit of the specific vehicle. If the required deceleration is within the vehicle's capability, the impact can be avoided.

In determining the required deceleration, we wish to find the minimum deceleration required to avoid the collision. That is, we don't wish to advise the driver to brake any harder than necessary to avoid the impact. Impact is avoided if the velocity of the trailing vehicle,  $V_1$ , is less than or equal to the velocity of the lead vehicle,  $V_2$ , when the distance between them has been closed. That is, impact is avoided if  $v_1(t) \leq v_2(t)$  when  $s_2(t) - s_1(t) = L_2$ . For minimum necessary braking, we want to find the deceleration that produces  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$ .

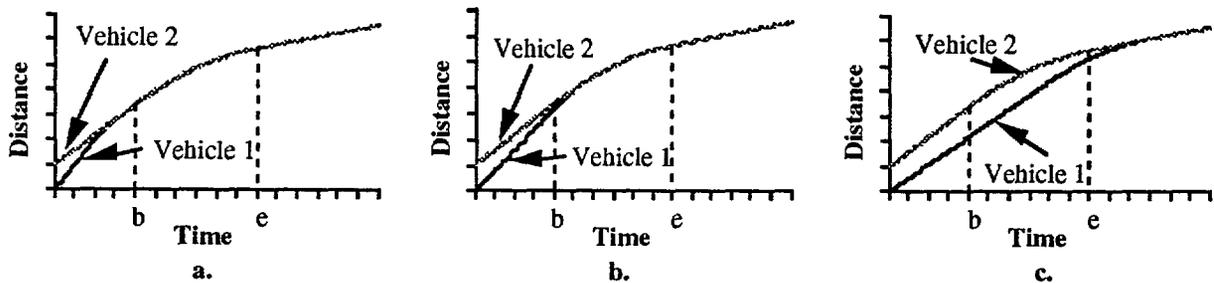


FIGURE 8a-c Three possible collision avoidance scenarios.

If we acknowledge, as in the previous section, that the lead vehicle may also be decelerating in response to the vehicle ahead of it, we have three possible collision avoidance scenarios shown in Figure 8a-c. In each of these cases,  $s_2(t) - s_1(t) = L_2$ , and  $v_1(t) = v_2(t)$  when the curves intersect. (In each of these, we have again simplified the figure, showing the trailing vehicle traveling at constant velocity prior to beginning deceleration, and the lead vehicle traveling at constant velocity before and after its deceleration. The equations to follow make no such assumptions.) In Figure 8a, the trailing vehicle decelerates such that  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$  before the lead vehicle begins its deceleration.

This situation can occur, for example, when the lead vehicle is "free-driving"; in other words, when the lead vehicle is not influenced by the vehicle ahead of it, or when the lead vehicle has a long reaction time prior to beginning its braking action. In Figure 8b,  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$  are reached while  $V_2$  is decelerating, and in Figure 8c,  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$  are reached after  $V_2$  has responded to the vehicle ahead of it.

If we again assume that vehicle  $V_2$  sustains a constant acceleration rate,  $a_2(0)$ , (not necessarily zero) between time 0 and  $b_2$ , a constant acceleration rate,  $a_2(b_2)$ , between time  $b_2$  and  $e_2$ , and a constant acceleration rate,  $a_2(e_2)$ , after time  $e_2$ , we can describe  $V_2$ 's position and velocity at time  $t$  by Equations 1 and 2 (as in Section 2.1.2.1).

$$v_2(t) = v_2(\tau_2) + a_2(\tau_2) \cdot (t - \tau_2) \quad (1)$$

$$s_2(t) = s_2(\tau_2) + v_2(\tau_2) \cdot (t - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (t - \tau_2)^2 \quad (2)$$

where,

$$\tau_2 = \begin{cases} 0, & t \leq b_2 \\ b_2, & b_2 < t \leq e_2 \\ e_2, & t > e_2 \end{cases}$$

In this case, since  $V_1$  decelerates to avoid the collision with  $V_2$ , we can describe the velocity and position of  $V_1$  as a function of  $t$  by Equations 5 and 6, where  $t > b_1$ , otherwise the vehicles collide prior to  $V_1$ 's deceleration (the scenario of the previous section).

$$v_1(t) = v_1(b_1) + a_1(b_1) \cdot (t - b_1) \quad (5)$$

$$s_1(t) = s_1(b_1) + v_1(b_1) \cdot (t - b_1) + \frac{1}{2} \cdot a_1(b_1) \cdot (t - b_1)^2 \quad (6)$$

These equations assume that the acceleration of  $V_1$  is a constant,  $a_1(0)$ , prior to the beginning of  $V_1$ 's deceleration at time  $b_1$ , and thus  $v_1(b_1)$  and  $s_1(b_1)$  are independent of  $t$ , and are given as

$$v_1(b_1) = v_1(0) + a_1(0) \cdot b_1, \text{ and}$$

$$s_1(b_1) = s_1(0) + v_1(0) \cdot b_1 + \frac{1}{2} \cdot a_1(0) \cdot b_1^2.$$

Our goal is to find the deceleration required by vehicle  $V_1$  once it begins its deceleration at time  $b_1$ . This deceleration is  $a_1(b_1)$  in Equations 5 and 6. Thus, we need to solve for  $a_1(b_1)$ , given the requirement that at some time  $t > b_1$ ,  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$ . Utilizing Equations 1, 2, 5, and 6, we solve (see Appendix) for the required deceleration  $a_1(b_1)$ , and the time,  $e_1$ , when the conditions  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$  are satisfied (Equations 7 and 8 correspond to equations A.11 and A.12 in the appendix).

$$a_1(b_1) = \frac{-[v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)]^2}{2 \cdot [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2]} + a_2(\tau_2) \quad (7)$$

$$e_1 = \frac{-2 \cdot [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2]}{v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)} + b_1 \quad (8)$$

Note that the bracketed term in the numerator of Equation 7 is the same as the denominator of Equation 8, and that the bracketed term in the denominator of Equation 7 is the same as the bracketed term in the numerator of Equation 8. If we let\*

$$\Delta V_{12}(\tau_2) = v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1), \text{ and}$$

---

\*  $\Delta V_{12}$  and  $\Delta S_{12}$  denote a difference in velocity and distance, respectively, between vehicles  $V_1$  and  $V_2$ .

$$\Delta S_{12}(\tau_2) = s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2,$$

we can rewrite Equations 7 and 8, producing Equations 9 and 10.

$$a_1(b_1) = \frac{-[\Delta V_{12}(\tau_2)]^2}{2 \cdot \Delta S_{12}(\tau_2)} + a_2(\tau_2) \quad (9)$$

$$e_1 = \frac{-2 \cdot \Delta S_{12}(\tau_2)}{\Delta V_{12}(\tau_2)} + b_1 \quad (10)$$

As in Section 2.1.2.1, the solution is based on  $\tau_2$ , which takes on a different value for each segment of the lead vehicle's distance-versus-time curve (see Figure 5). In order to determine in which segment of that curve the conditions  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$  are satisfied, we employ the same procedure we used in Section 2.1.2.1. To simplify the following, we will say the vehicles "meet" when the conditions  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$  are satisfied. To determine the correct values for  $a_1(b_1)$  and  $e_1$ , we first assume the vehicles meet prior to  $b_2$ . That implies that  $\tau_2 = 0$  in the expression for  $e_1$ . We then solve Equation 10 for  $e_1$ . If the resulting value for  $e_1$  is in fact less than  $b_2$ , then the vehicles meet at time  $e_1$ , and the deceleration required is given by substituting  $\tau_2 = 0$  into Equation 9. If the solution value for  $e_1$  is greater than  $b_2$ , then the vehicles meet some time after  $b_2$ . We then assume that the vehicles meet between time  $b_2$  and  $e_2$ , letting  $\tau_2 = b_2$ . We again solve for  $e_1$  and check if it falls between  $b_2$  and  $e_2$ . If it does, we determine the required deceleration from Equation 9. If not, we assume the vehicles meet after  $e_2$ . Letting  $\tau_2 = e_2$  in Equation 10, we solve for  $e_1$ . If  $e_1 > e_2$ , the vehicles meet after  $e_2$ , otherwise the vehicles will not meet given the current conditions (e.g. the lead vehicle is pulling away from the trailing vehicle), and no deceleration is required. If no deceleration is required, we will assume that  $V_1$  will continue at its current acceleration,  $a_1(0)$ . We can produce this assumption by letting  $a_1(b_1) = a_1(0)$ , and giving  $e_1$  a very large value, which we will designate by  $\infty$  (i.e. the vehicle doesn't change acceleration).

We can also determine the acceleration,  $a_1(e_1)$ , for vehicle  $V_1$  at the end of the required deceleration. We assume that once the conditions  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$  are satisfied at time  $e_1$ , the trailing vehicle,  $V_1$ , maintains its distance to the lead vehicle,  $V_2$ , following the lead vehicle's distance-versus-time curve by taking on the lead vehicle's acceleration.

The procedure for determining  $a_1(b_1)$ ,  $e_1$ , and  $a_1(e_1)$  is shown using a pseudo-programming language in Figure 9. The program segment utilizes the distance-versus-time curve for the lead vehicle, which is given by the 7-tuple  $(s_2(0), v_2(0), a_2(0), b_2, a_2(b_2), e_2, a_2(e_2))$ , the length of the lead vehicle,  $L_2$ , and the current state of the trailing vehicle, which is given by the quadruple  $(s_1(0), v_1(0), a_1(0), b_1)$ , to produce the required values for the remainder of the trailing vehicles distance-versus-time curve given by the triple  $(a_1(b_1), e_1, a_1(e_1))$ .

**if**  $\left( 0 \leq \frac{-2 \cdot \Delta S_{12}(0)}{\Delta V_{12}(0)} + b_1 \leq b_2 \right)$  **then**

$$a_1(b_1) = \frac{-[\Delta V_{12}(0)]^2}{2 \cdot \Delta S_{12}(0)} + a_2(0);$$

$$e_1 = \frac{-2 \cdot \Delta S_{12}(0)}{\Delta V_{12}(0)} + b_1;$$

$$a_1(e_1) = a_2(0);$$

**else if**  $\left( b_2 < \frac{-2 \cdot \Delta S_{12}(b_2)}{\Delta V_{12}(b_2)} + b_1 \leq e_2 \right)$  **then**

$$a_1(b_1) = \frac{-[\Delta V_{12}(b_2)]^2}{2 \cdot \Delta S_{12}(b_2)} + a_2(b_2);$$

$$e_1 = \frac{-2 \cdot \Delta S_{12}(b_2)}{\Delta V_{12}(b_2)} + b_1;$$

$$a_1(e_1) = a_2(b_2);$$

**else if**  $\left( e_2 < \frac{-2 \cdot \Delta S_{12}(e_2)}{\Delta V_{12}(e_2)} + b_1 \right)$  **then**

$$a_1(b_1) = \frac{-[\Delta V_{12}(e_2)]^2}{2 \cdot \Delta S_{12}(e_2)} + a_2(e_2);$$

$$e_1 = \frac{-2 \cdot \Delta S_{12}(e_2)}{\Delta V_{12}(e_2)} + b_1;$$

$$a_1(e_1) = a_2(e_2);$$

**else**

$$a_1(b_1) = a_1(0);$$

$$e_1 = \infty;$$

$$a_1(e_1) = a_1(0);$$

**FIGURE 9** Pseudo-programming language code for determining required deceleration.

### 2.1.3 Multi-Vehicle Processing

In this section, we explain how the results from the previous section can be used to determine the correct action for a vehicle given information from multiple vehicles ahead (Figure 10). To determine the required deceleration for vehicle  $V_1$ , we utilize data from vehicles  $V_2$  through  $V_n$  ahead. These are the vehicles ahead of  $V_1$ , in the same lane, and within  $V_1$ 's communication range. From each of these vehicles,  $V_1$  receives its position, velocity, acceleration, and the expected time until any necessary braking action will occur,  $b$ . As mentioned above,  $b$  is based on its driver's expected reaction time and the state of the warning system.

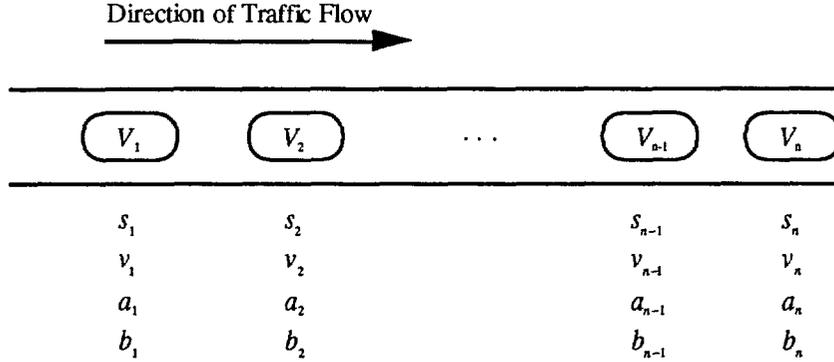


FIGURE 10 Multiple vehicles traveling in a single lane.

The previous section's result is that, for a pair of vehicles, given the distance-versus-time curve of the lead vehicle, the distance-versus-time curve for the trailing vehicle can be determined. The procedure shown in Figure 9 can be easily generalized such that for a given lead vehicle,  $V_{i+1}$ , we can determine the required deceleration of the trailing vehicle  $V_i$ . That is, from distance-versus-time curve for  $V_{i+1}$ , which is given by  $(s_{i+1}(0), v_{i+1}(0), a_{i+1}(0), b_{i+1}, a_{i+1}(b_{i+1}), e_{i+1}, a_{i+1}(e_{i+1}))$ , the length of the lead vehicle,  $L_{i+1}$ , and the current state of  $V_i$ , which is given by  $(s_i(0), v_i(0), a_i(0), b_i)$ , the remainder of the trailing vehicle's distance-versus-time curve,  $(a_i(b_i), e_i, a_i(e_i))$ , can be found. The current position, velocity, acceleration, and length of each vehicle is obtained via vehicle-to-vehicle communication. Also, we assume that each vehicle's system has an estimate of its own value for  $b$ , based on its driver's expected reaction time and the state of the warning system, which it communicates to other vehicles. Thus, our generalized program segment takes as input  $s_i(0), v_i(0), a_i(0), b_i, L_{i+1}, s_{i+1}(0), v_{i+1}(0), a_{i+1}(0), b_{i+1}, a_{i+1}(b_{i+1}), e_{i+1}$ , and  $a_{i+1}(e_{i+1})$ , and yields as a result  $a_i(b_i), e_i$ , and  $a_i(e_i)$ . We can define a function,  $\Phi$ , based on the program segment above which is a function of the 12-tuple  $(s_i(0), v_i(0), a_i(0), b_i, L_{i+1}, s_{i+1}(0), v_{i+1}(0), a_{i+1}(0), b_{i+1}, a_{i+1}(b_{i+1}), e_{i+1}, a_{i+1}(e_{i+1}))$ , and produces as a result the triple  $(a_i(b_i), e_i, a_i(e_i))$  (Equation 11).

$$(a_i(b_i), e_i, a_i(e_i)) = \Phi(s_i(0), v_i(0), a_i(0), b_i, L_{i+1}, s_{i+1}(0), v_{i+1}(0), a_{i+1}(0), b_{i+1}, a_{i+1}(b_{i+1}), e_{i+1}, a_{i+1}(e_{i+1})) \quad (11)$$

$\Phi$  in Equation 11 is a recursive function. The last three parameters to  $\Phi$ ,  $a_{i+1}(b_{i+1}), e_{i+1}$ , and  $a_{i+1}(e_{i+1})$ , are obtained by evaluating  $\Phi$  using data from vehicles  $V_{i+1}$  and  $V_{i+2}$ . This evaluation using the data from vehicles  $V_{i+1}$  and  $V_{i+2}$  will, in turn, require  $\Phi$  to be evaluated using data from vehicles  $V_{i+2}$  and  $V_{i+3}$ , etc. In order to define  $\Phi$ , we need to establish a termination condition to halt the recursion.

As the recursion continues, we will eventually need to generate values for  $a_n(b_n), e_n$ , and  $a_n(e_n)$ . Since we have no information from the vehicle ahead of  $V_n$  (if one is present), we have to make an assumption about the distance-versus-time curve for  $V_n$ . We will assume that  $V_n$  will continue at its current acceleration,  $a_n(0)$ . We can produce this assumption by letting  $a_n(b_n) = a_n(0)$ , and giving  $e_n$  a very large value, designated below by  $\infty$  (i.e. the vehicle doesn't change acceleration). This is similar to the case where no deceleration is required, discussed earlier, but is forced by a lack of information about the lead vehicle. Using this termination condition, we can now define  $\Phi$ .

$$\begin{aligned}
& (a_i(b_i), e_i, a_i(e_i)) \\
& = \begin{cases} (a_n(0), \infty, a_n(0)), & i = n \\ \Phi(s_i(0), v_i(0), a_i(0), b_i, L_{i+1}, s_{i+1}(0), v_{i+1}(0), a_{i+1}(0), b_{i+1}, a_{i+1}(b_{i+1}), e_{i+1}, a_{i+1}(e_{i+1})), & i < n \end{cases} \quad (12)
\end{aligned}$$

Utilizing Equation 12, we determine  $a_i(b_i)$ ,  $e_i$ , and  $a_i(e_i)$  for vehicle  $V_i$ .

$$\begin{aligned}
& (a_i(b_i), e_i, a_i(e_i)) = \\
& \Phi(s_1(0), v_1(0), a_1(0), b_1, L_2, s_2(0), v_2(0), a_2(0), b_2, \\
& \Phi(s_2(0), v_2(0), a_2(0), b_2, L_3, s_3(0), v_3(0), a_3(0), b_3, \\
& \Phi(s_3(0), v_3(0), a_3(0), b_3, L_4, s_4(0), v_4(0), a_4(0), b_4, \\
& \dots \\
& \Phi(s_{n-3}(0), v_{n-3}(0), a_{n-3}(0), b_{n-3}, L_{n-2}, s_{n-2}(0), v_{n-2}(0), a_{n-2}(0), b_{n-2}, \\
& \Phi(s_{n-2}(0), v_{n-2}(0), a_{n-2}(0), b_{n-2}, L_{n-1}, s_{n-1}(0), v_{n-1}(0), a_{n-1}(0), b_{n-1}, \\
& \Phi(s_{n-1}(0), v_{n-1}(0), a_{n-1}(0), b_{n-1}, L_n, s_n(0), v_n(0), a_n(0), b_n, a_n(0), \infty, a_n(0)) \dots) \quad (13)
\end{aligned}$$

Figure 11 is a graphic depiction of Equation 13. The arrows labeled  $\Phi$  indicate the application of the function  $\Phi$  with the solid-line-enclosed values at the tail of the arrow as input, and the dash-enclosed values at the head of the arrow as the result.

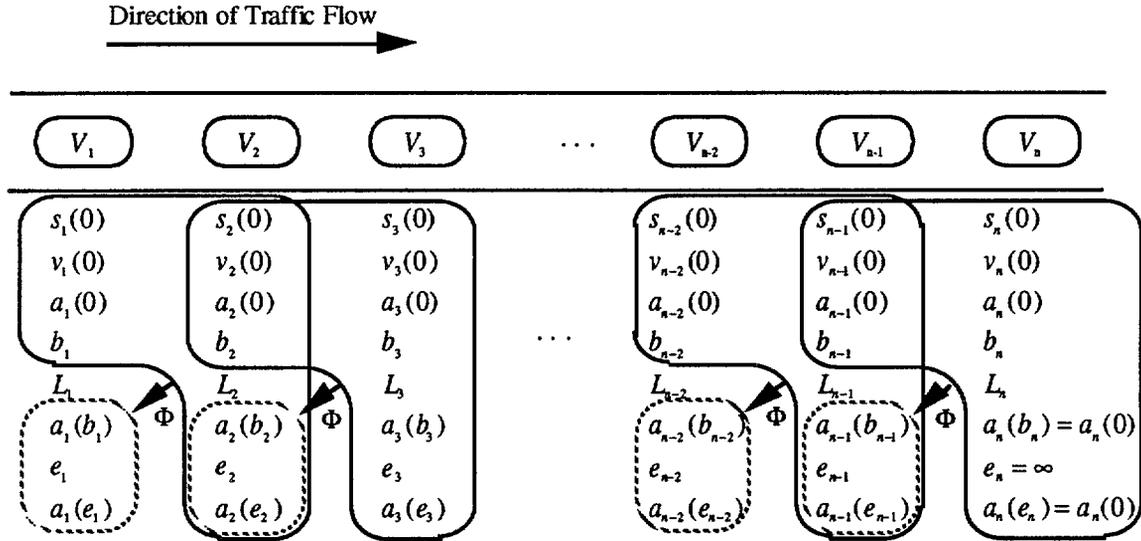


FIGURE 11 Multiple vehicles traveling in a single lane.

### 2.1.4 Warning Device

For the purposes of this report, we will assume the in-vehicle warning device to be a five segment light display where increasing levels of required deceleration are indicated by an increasing number of illuminated lights on the display, possibly accompanied by an auditory warning. The level of the display indicates the urgency of the braking action required.

The in-vehicle warning device is based on the required deceleration,  $a_i(b_i)$ , if any, as obtained from Equation 13. Since research indicates that a comfortable rate of deceleration for the average driver, for example when decelerating to a stop, is typically 30% of the maximum deceleration of which the vehicle is capable, for large distance headways (greater than 150 ft.), the threshold for the device is set at 30% of the maximum deceleration. Thus, the warning device is

activated when the required deceleration,  $a_1(b_1)$ , is between 30% of the maximum deceleration, and the maximum deceleration achievable for the vehicle. For a 5 segment display, one light is illuminated when the required deceleration is between 30% and 44% of the maximum deceleration, two lights when the required deceleration is between 44% and 58% three lights when the required deceleration is between 58% and 72%, four lights when the required deceleration is between 72% and 86%, and five lights are illuminated when the required deceleration is between 86% and 100% of the maximum deceleration.

At shorter distance headways (150 ft. or less), the sensitivity of the warning device is increased linearly. For example, at 75 ft., the device is activated when a deceleration of greater than 15% of the vehicle's maximum deceleration is required.

The calculation of the required deceleration based on the current data received is repeated often, in the experiments below every 10 ms, and the on-board warning state is changed appropriately. Each equipped vehicle in the system does similar processing using the (most likely different) data that it receives from nearby equipped vehicles. The warning device proposed here is an example of the type of device that may be used. The actual configuration and integration of the device will require a separate investigation.

## 2.2 ALGORITHM EVALUATION STRATEGY

To this point, we have developed an algorithm which yields the deceleration required by a vehicle in response to several vehicles ahead of it. To evaluate the potential benefit of the system, we initially assumed 100% system penetration, and studied the impact of the following variables on the frequency and severity of expected collisions, and the braking force required to avert a collision:

- number of vehicles ahead from which to consider data
- communication range
- severity of lead vehicle braking action
- variation in driver reaction times from the expected value
- variation in degree of response to the warning device
- assumptions about the behavior of the furthest vehicle ahead

We then studied the issue of deployment by examining the frequency and severity of expected collisions, and the braking force required to avert a collision for various levels of system penetration, and for various levels of driver reliance of the warning device.

The following sections describe the experimental setup used, and our strategy for evaluating the impact of the above variables for both 100% system penetration, and during deployment

### 2.2.1 Experimental Setup

We evaluated the potential impact of the system using computer simulation. This simulation, however, is based on actual highway data collected by the FHWA\*. The data used was comprised of two sets of data, each representing a day of traffic data collected on Interstate-40 in New Mexico. The first set of data was collected on September 25, 1991, and consists of data from 36,342 vehicles. The second set of data was collected on July 11, 1993, and consists of data from 31,612 vehicles. Each set of data contains the velocity of each vehicle, and its headway to the preceding vehicle.

To study the impact of the system, we generated an incident by assuming a more or less severe deceleration for a given vehicle in a chain of vehicles. We studied the degree to which the system is effective in 1) reducing the number of collisions, 2) reducing the impact of collisions which do occur, and 3) reducing the required braking force when a collision is averted. We chose these three factors for the following reasons. First, reducing the number of collisions has obvious benefits in terms of reducing injuries and vehicle damage, as well as reducing congestion and delay which accidents produce. Second, reducing the impact of collisions which do occur also translates to reductions in injuries and vehicle damage due to the reduction in the force of the collision. Finally, reducing the braking force when a collision is averted adds to driver comfort, and it also helps to reduce the amount of disturbance introduced in the vehicles following the decelerating vehicle.

Since the set of vehicles for a given day is large, we divided each set into smaller groups or "clusters" of vehicles so that experiments could be run on a cluster of vehicles without considering the remainder of the vehicles in the data set. To do this we chose to distinguish one cluster from another by the distance between vehicles in the cluster. Thus, we define a cluster to be a group of vehicles in a single lane where the distance between any pair of vehicles in the group is

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\* Data provided by Eugene Farber, Ford Motor Co.

less than or equal to some distance,  $S_{IC}$ , the intra-cluster spacing, and the distances between the lead vehicle in the group and the vehicle ahead of it, and the trailing vehicle in the group and the vehicle behind it are each greater than  $S_{IC}$ . Figure 12 depicts this relationship. If there is a large enough distance between the vehicles in the cluster and the other vehicles on the roadway, we assume that the vehicles in the cluster act independently of the other vehicles on the roadway, and thus those vehicles need not be considered when studying the vehicles in the cluster.

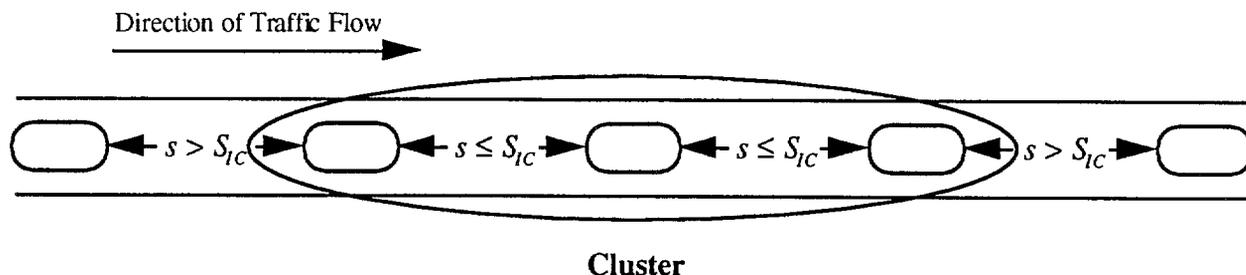


FIGURE 12 Definition of a cluster of vehicles.

For each data set, we chose an intra-cluster spacing, and pre-processed the data, dividing the raw data into a set of clusters. During this step, we discarded clusters of size one and two, since we are interested in utilizing data from multiple vehicles ahead of a particular vehicle. Tables 1 and 2 show the results of this pre-processing on the September 25, and the data July 11 files, respectively.

Intra-Cluster Spacing (ft.)	Number of Clusters	Cluster Size			Average Cluster Length (ft.)	Average Cluster Separation (ft.)
		Minimum	Maximum	Average		
300	3654	3	427	7.91	750	620
400	3229	3	427	9.67	1099	744
500	2825	3	482	11.55	1502	869
600	2435	3	482	13.72	1976	971
700	2098	3	482	16.23	2550	1069
800	1819	3	482	18.94	3190	1165
900	1657	3	482	20.98	3718	1255

TABLE 1 September 25 data file's cluster characteristics for various intra-cluster spacings.

Intra-Cluster Spacing (ft.)	Number of Clusters	Cluster Size			Average Cluster Length (ft.)	Average Cluster Separation (ft.)
		Minimum	Maximum	Average		
300	4129	3	66	5.93	635	538
400	3714	3	132	7.49	1001	642
500	3117	3	171	9.47	1475	724
600	2509	3	524	12.15	2134	793
700	1965	3	524	15.74	3020	849
800	1590	3	524	19.64	4020	880
900	1285	3	524	24.40	5247	982

TABLE 2 July 11 data file's cluster characteristics for various intra-cluster spacings.

The intra-cluster spacing used for a particular experiment will be discussed in the sections to follow.

Thus, each cluster obtained from the raw data consists of three or more vehicles, and for each, the velocity and the separation between adjacent vehicles is known. To isolate the effects of the system, independent of individual vehicle characteristics, we assume a homogeneous set of passenger vehicles in terms of size and maximum deceleration. In

addition, we need to make assumptions about the acceleration of each vehicle in the cluster at the beginning of the experiment, and the reaction time of each driver.

For each cluster, we assume random reaction times for each driver in the group drawn from an appropriate distribution. The distribution used is a lognormal distribution with a mean of 1.21 seconds and a standard deviation of 0.63 seconds. This distribution is based on data collected by Sivak et al. (20) for unalerted drivers following a test vehicle, while being monitored by a trailing vehicle. They collected 1,644 reaction times recording only those of three seconds or less. In generating the random reaction time for each driver, we chose values between their 5th and 95th percentile. Thus, each random reaction time is drawn from the lognormal distribution above, and has a minimum value of 0.48 seconds and a maximum value of 2.40 seconds. Figure 13 shows the distribution used. We assume that the extremely long reaction times, often attributed to driver inattention, will be eliminated by the alert provided by the warning system.

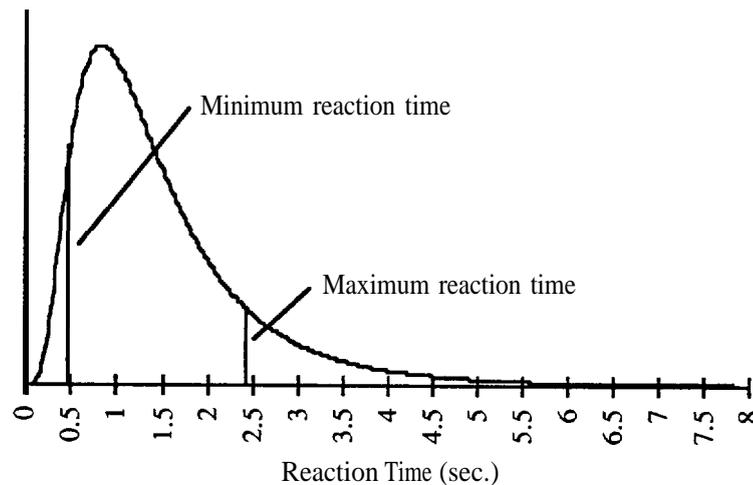


FIGURE 13 Driver reaction time distribution.

Finally, we need to establish initial acceleration rates for each vehicle in a cluster. Since the recorded data contains no acceleration data we will assume an initial acceleration rate of zero for each vehicle in the cluster.

These assumptions combined with recorded data define the initial state for a cluster. It is possible that the randomly drawn reaction times and the zero acceleration rate assumption may produce an initial state which places the cluster in an invalid state. We define a cluster's initial state to be invalid if the warning device in any vehicle in the cluster is already active, or if any vehicle is within the minimum desired distance to the vehicle ahead (two vehicle lengths, for these experiments) in the initial state. Any clusters that have an invalid initial state based on the parameters selected are discarded.

In the experiments discussed below, we generate an artificial incident by assuming that the lead vehicle in the cluster decelerates to a stop, with a constant deceleration rate. The simulation is a discrete time simulation in which the state of each vehicle is updated every 10 ms. At each 10 ms time step, we update the position and velocity of each vehicle. At each time step, we also update the state of each equipped vehicle's warning device, the driver's remaining time to braking, and the vehicle's acceleration rate once the braking action begins. We allow the simulation to run until all vehicles in the cluster have come to a stop. During the run, we accumulate the number and severity of collisions, and the braking force required in the event of no collision.

### 2.2.2 100% System Penetration

First, we studied the case of 100% system penetration. Here, each vehicle in the system is equipped and operating properly. Under this assumption, we studied the impact of the following variables on the frequency and severity of expected collisions, and the braking force required to avert a collision:

- number of vehicles ahead from which to consider data
- communication range
- severity of lead vehicle braking action
- variation in driver reaction times from the expected value

- variation in degree of response to the warning device
- assumptions about the behavior of the furthest vehicle ahead

In this section, we discuss the strategy used to study these variables. In the next section, we consider the case of less than 100% system penetration.

#### *2.2.2.1 Number of Vehicles Ahead from which to Consider Data*

During operation, the communication range will provide a limit on the number of vehicles ahead from which a given vehicle can receive data. This is a physical limit based on the distance ahead from which we can receive data, and the positions of the vehicles in the environment. In addition to this, it may be desirable to place a limit within the algorithm on the number of vehicles from which to process data. This will help to provide a bound on the processing time and computation facility required, as well as eliminating unnecessary processing of data from vehicles which may be irrelevant. In this evaluation, we determine how the number and severity of incidents, and the required braking force are impacted by varying the number of vehicles from which data is utilized. From this, we can determine an appropriate limit on the number of vehicles to consider in the system. The results of this experiment are discussed in Section 3.1.1.

#### *2.2.2.2 Communication Range*

The communication range directly effects how far a vehicle must be from the vehicle ahead in order to receive information from that vehicle, and thus how early a warning can be delivered. A very short communication range is comparable to an unequipped driver operating in a fog situation, where knowledge of the vehicle(s) ahead is available only at short headways, and thus the chance of an accident occurring is increased.

It appears so far that the ability to communicate with vehicles at a greater distance, and thus with greater time to adjust to the vehicle ahead, will reduce incidents. It also appears that beyond some point as the communication range is increased, the incremental benefit will diminish. In this evaluation, we determine how the number and severity of incidents, and the braking force required to avert a collision are impacted by varying the communication range, and thus determine an appropriate communication range for use in the system. The results of this experiment are discussed in Section 3.1.2.

#### *2.2.2.3 Severity of Lead Vehicle Braking Action*

In the experiments discussed above, we have utilized a random lead vehicle deceleration uniformly distributed between 30% of the maximum deceleration, and the maximum deceleration possible for the lead vehicle. Obviously, the more severe the deceleration of the lead vehicle, the more likely it is that a collision will occur. In this evaluation, we studied how the benefits of the collision warning system are affected by severe lead vehicle deceleration. We repeated the experiments above with the lead vehicle deceleration equal to the maximum deceleration possible for the lead vehicle. The results of this experiment are discussed in Section 3.1.3.

#### *2.2.2.4 Variation in Driver Reaction Times from the Expected Value*

For a given cluster in which we generate an incident, we assign random reaction times for each driver. We assume that during actual operation, the on-board software will have an estimate of the driver's reaction time. This estimate could be a fixed value, or could be learned over time for a given driver. For a particular braking event, it is likely that the driver's actual reaction time will be different from the value assumed by the software. To study the effect of these differences, we chose, for each driver, a random reaction time which represents the expected reaction time assumed by the system, and a random variation from that time, which will represent the driver's actual response to the given event. We then examined how variations in actual reaction time from the expected reaction time impact the collision warning system. The results of this experiment are discussed in Section 3.1.4 and 3.1.6.

#### *2.2.2.5 Variation in Degree of Response to the Warning Device*

During operation, the on-board system in each vehicle will determine any required deceleration for that vehicle. The system then provides this advice to the driver. Regardless of the specifics of the man-machine interface which provides this advice, the driver will have to associate a particular warning with a corresponding braking action. Thus, it

appears likely that the response required by the vehicle, and that provided by the driver will vary to some degree for any particular warning. In this evaluation, we study how variations in the degree of response to the warning device impact the effectiveness of the collision warning system. The results of this experiment are discussed in Section 3.1.5 and 3.1.6.

#### 2.2.2.6 Assumptions about the Behavior of the Furthest Vehicle Ahead

In a set of vehicles, each vehicle's system will provide a warning (if necessary) based on data from a limited number of vehicles ahead. The number of vehicles ahead from which a given vehicle utilizes data will be a function of the vehicle's communication range, and the algorithm's vehicle limit (as discussed in Section 2.2). In any case, the algorithm provides a warning based on a limited number of vehicles. The algorithm calculates the necessary deceleration for a given vehicle in a chain of vehicles based on the current state of that vehicle and the vehicle ahead, and the expected response of the vehicle ahead. The expected response of the vehicle ahead is based on the vehicles ahead of it, and so on.

The issue here is to establish the expected behavior for the furthest vehicle ahead from which data has been received. Since we have no data from any vehicles ahead of this vehicle, we need to make an assumption about the behavior of this vehicle. We might assume, for example, worst case behavior for this vehicle. That is, we might assume that the furthest vehicle ahead from which we have data is about to decelerate with some maximum deceleration rate. On the other hand, we might assume a default expected behavior for the lead vehicle. That is, the lead vehicle will continue accelerating or decelerating at its current rate over the next time interval. In this evaluation, we intend to determine how varying assumptions about the furthest vehicle ahead impact the calculated response for the vehicle in question, and the number and severity of collisions expected. From this, we will establish the appropriate assumption(s) to be utilized in the system. This is discussed further in Section 3.1.7.

### 22.3 System Deployment

In this section, we discuss the strategy for determining the benefits of the system at various levels of system penetration. We make the following assumptions about system deployment:

- each equipped vehicle has a sensor which provides the position, velocity, and acceleration of the vehicle immediately ahead
- level of penetration increases from 0% to 100% over time [0% 10%. 20%. . . . 100%]
- driver has perfect visual information for 1, 2, or 3 vehicles ahead (position, velocity, acceleration, and driver reaction time)
- driver reliance ( $r$ ) on the system varies from 0 to 1 [0.0, 0.1, 0.2, . . . . 1.01]
- driver uses a weighted average of visually determined and system recommended deceleration values, using  $r$  as the weight.

In order to vary the level of system penetration in the simulation, we used the same basic setup discussed in Section 2.2.1, but we made several modifications.

We assume that each equipped vehicle has a sensor which provides information about the vehicle immediately ahead. If the vehicle immediately ahead is also equipped, the data provided is redundant. However, it is important that we have the ability to obtain data from an unequipped vehicle immediately ahead. Without that data, the warning provided by the system will be insufficient to avoid a collision with the vehicle ahead because the system would have no information about that vehicle, and thus not respond to any braking action that the vehicle takes. We believe that this assumption is (or soon will be) valid, given the expected diffusion of "intelligent cruise control," which may be expected to have such sensors.

For a given level,  $x\%$ , of penetration each vehicle in a cluster is randomly selected as an equipped vehicle with a probability of  $x\%$ . Thus, the entire data set consists of approximately  $X\%$  equipped vehicles. We studied penetration levels from 0% to 100% in increments of 10%.

In the case of 100% penetration reported above, we assumed that the driver performed any required braking in response to the warning device only. Under that assumption, we didn't need to consider the characteristics of the driver of an unequipped vehicle. As we consider the case of less than 100% penetration, we need to model the unequipped driver. We can then vary the percentage of equipped and unequipped drivers in the simulation, and compare the results.

In modeling the unequipped driver, we assume the unequipped driver has perfect information about the vehicles ahead which the driver can see. That is, the driver has accurate knowledge of the position, velocity, acceleration, and driver reaction time for the vehicles within the driver's range of vision. We assume that the typical driver can see at most three vehicles ahead under normal conditions (21). We performed separate simulations assuming the unequipped driver had the ability to see 1, 2, or 3 vehicles ahead.

We also assume that each equipped driver in the system has the same visual ability as the unequipped drivers, but

has the additional information provided by the warning device. We assume that the driver's reliance ( $r$ ) on the warning device varies from 0 to 1. A reliance value of zero means that the driver disregards the warning device, and thus responds like an unequipped driver. A reliance value of one means that the driver responds only to the warning device, and disregards visual information ("driving by instruments"). For reliance values between 0 and 1, we assume a response ( $a_r$ ) equal to the weighted average of the visually determined ( $a_1$ ) and the system recommended ( $a_2$ ) deceleration values. Thus,  $a_r = a_1(1 - r) + a_2r$ . We examined values of  $r$  from 0 to 1 by increments of 0.1.

### 3.0 RESULTS

#### 3.1 100% SYSTEM PENETRATION

In this section, we assume throughout that there is 100% penetration of the system under consideration. Deployment issues will be considered below.

##### 3.1.1 Number of Vehicles Ahead from which to Consider Data

In this case, we wish to determine the number of vehicles ahead from which data is useful. We assume that there will be diminishing benefit as more and more vehicles ahead are considered. The number of vehicles ahead from which to consider data, or the look-ahead number, is closely related to the vehicle's communication range. The communication range places a physical limit on the possible number of vehicles from which data can be received. If, for example, we chose a look-ahead number of 10, but the communication range was such that 10 vehicles were never in range then the effective look-ahead number would be less than 10. To isolate the look-ahead number from the communication range, we chose a very large communication range for this experiment. With an essentially infinite communication range, the look-ahead number is independent of the communication range.

As we have stated, we generate an incident for a particular cluster by producing a more or less severe deceleration in the lead vehicle in the cluster. For this experiment, we will choose a random lead vehicle deceleration uniformly distributed between a moderately severe deceleration, 30% of the maximum deceleration, and the worst case deceleration, the maximum deceleration possible for the lead vehicle. Later, we will examine the effects of variations in the lead vehicle deceleration on the system.

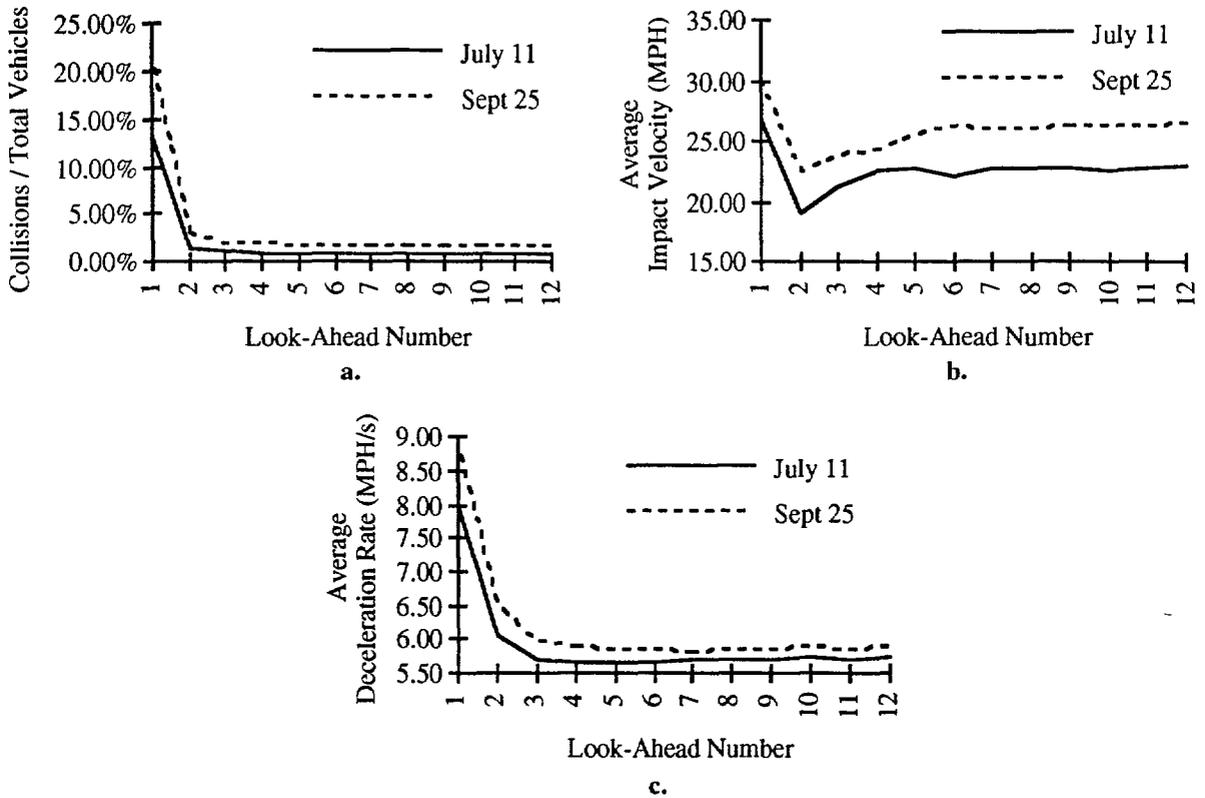
To further isolate the look-ahead number, for this experiment, we also assume no variation in driver reaction time from the expected value (see Section 2.2.2.5), accurate driver response to the warning system (see Section 2.2.2.6), and the default expected behavior for the lead vehicle (see Section 2.2.2.7).

With these parameters fixed, we vary the look-ahead number, and examine the impact on the number and severity of collisions, and the braking force required in the event of no collision. For a given cluster, we choose random reaction times for each driver, and a random deceleration rate for the lead vehicle. We then vary the look-ahead number, but for each look-ahead number we use the same reaction times, and lead vehicle deceleration rate. We do this for each cluster in both data sets, and compare the number and severity of collisions, and the braking force required when there is no collision, for each look-ahead number selected.

As a starting point, we assume that ideal look-ahead number is less than or equal to 12, which limits the number of experiments to run. This implies that using a look-ahead number of 12 as opposed to 11 in the system may reduce the number of collisions, impact velocity, or deceleration rate. Since there is no way to discern this possible gain in clusters of size less than size 13 (looking ahead 12 is the same as 11 in a cluster of size 5, for example), we initially chose only clusters larger than 12 vehicles. To get the most clusters of this size, we chose the intra-cluster spacing which provided the largest number of clusters greater than 12 vehicles. After examining the number of clusters greater than 12 vehicles for various intra-cluster spacings, we found that an intra-cluster spacing of 600 ft. and 700 ft. produced the most clusters greater than size 12 from the September 25 and July 11 data files, respectively. The September 25 data set yielded 300 clusters, and the July 11 data set yielded 527 clusters using these intra-cluster spacings. Figure 14 shows the impact of varying look-ahead numbers on these sets of clusters.

Figure 14a shows that increasing the look-ahead number reduces the percentage of vehicles that experience a collision in each data set. This result indicates that very little or no benefit is obtained by utilizing data from more than 6 or 7 vehicles ahead. Figure 14c shows a similar result in terms of the average deceleration rate required by the vehicles not involved in collisions. In Figure 14b, the average impact velocity of the vehicles involved in collisions drops rapidly initially, but the climbs as we increase the look-ahead number. This occurs because higher impact collisions are eliminated first followed by the lower impact collisions. This results in the average decreasing quickly and then returning to a stable level as fewer and fewer collisions are eliminated.

We should also note that the differences between the two independent data sets is caused by the differences in the characteristics of the traffic flow for each day. The September 25 traffic had a slower average speed, but shorter average time headway to the vehicle ahead (22).



**FIGURE 14 Impact of look-ahead number for clusters greater than 12 vehicles.**

We also performed the same experiment utilizing the same intra-cluster spacings, but including all the clusters, and not just those larger than 12 vehicles. Including all clusters greater than 2 vehicles, the September 25 data set yielded 1761 clusters, and the July 11 data set yielded 1522 clusters. Figure 15 shows the impact of varying look-ahead numbers on these sets of clusters.

The results of Figure 15 again indicate that there is significant gain in terms of reduction in the number and severity of incidents, and the braking force required to avert a collision by looking ahead 6 or 7 vehicles, but that very little or no benefit is obtained by utilizing data from more than 6 or 7 vehicles ahead.

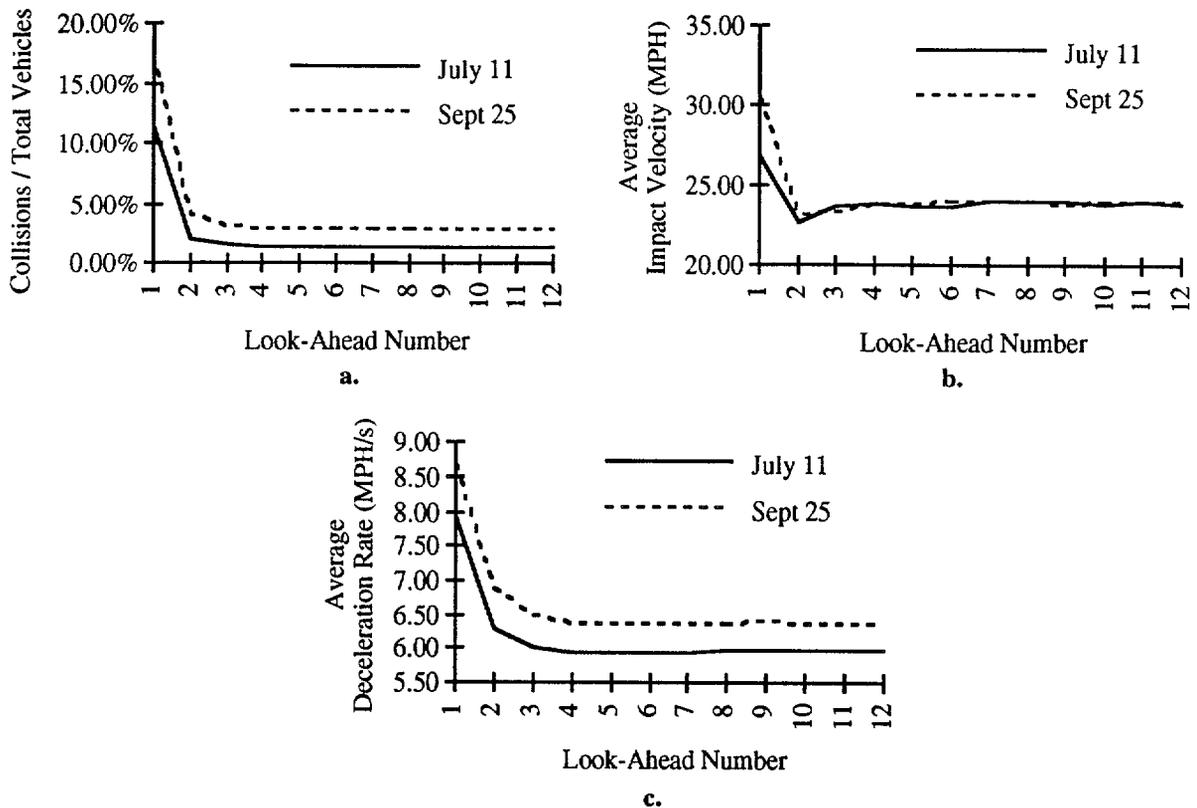


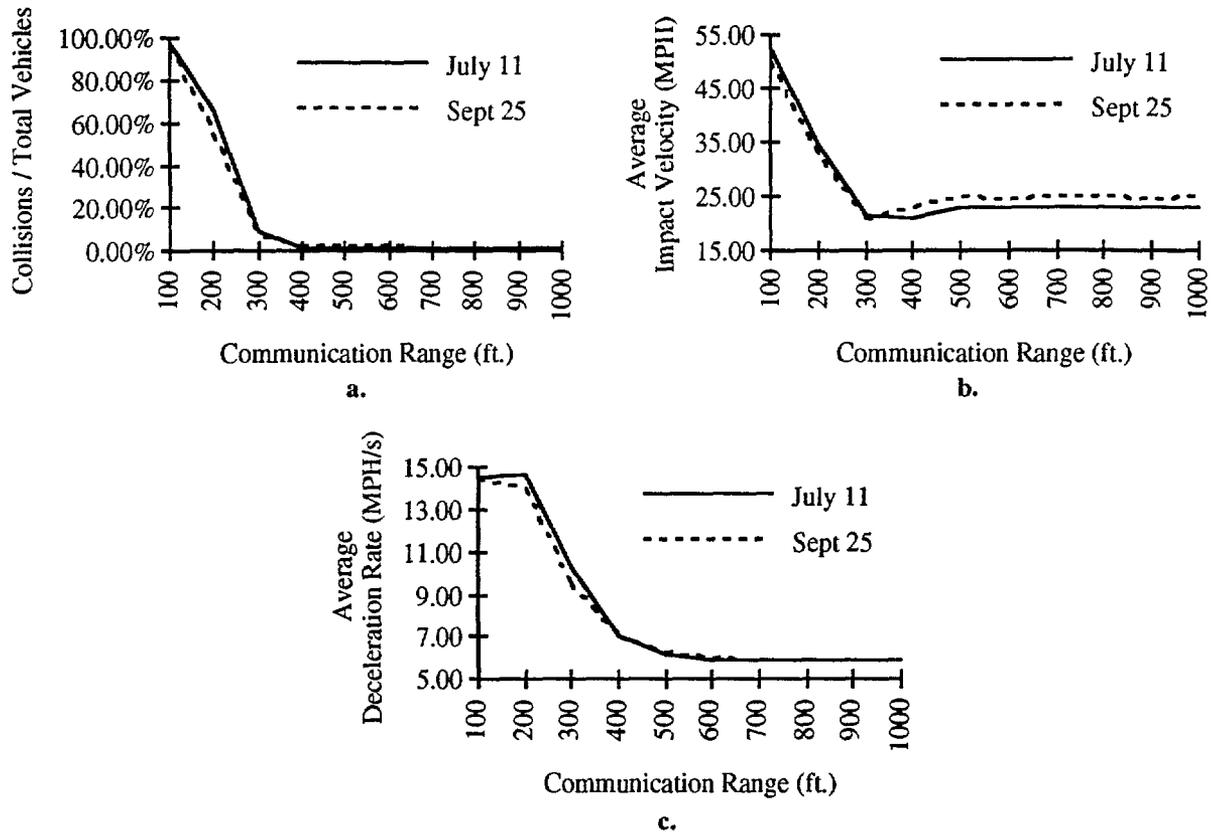
FIGURE 15 Impact of look-ahead number for clusters greater than 2 vehicles.

### 3.1.2 Communication Range

In this experiment, we utilize the results of the previous experiment, and fix the look-ahead number at 7 vehicles. In addition, as in the above experiment, we choose a lead vehicle deceleration between 30% of the maximum deceleration, and the maximum deceleration possible for the lead vehicle. We also assume, as above, no variation in driver reaction time from the expected value, accurate driver response to the warning system, and the default expected behavior for the lead vehicle.

With these parameters fixed, we vary the communication range, and examine the impact on the number and severity of collisions, and the braking force required when no collision occurs. For a given cluster, we choose random reaction times for each driver, and a random deceleration rate for the lead vehicle. We then vary the communication range, but for each communication range we use the same reaction times, and lead vehicle deceleration rate. We do this for each cluster in both data sets, and compare the number and severity of collisions, and the braking force required for non-collisions, for each communication range selected.

To limit the number of experiments, we examined communication ranges up to 1000 ft., in increments of 100 ft. Since we have fixed the look-ahead number at 7 vehicles, we chose only clusters larger than 7 vehicles, for the same reasons as in the previous section. To get the most clusters of this size, we chose the intra-cluster spacing which provided the largest number of clusters greater than 7 vehicles. After examining the number of clusters greater than 7 vehicles for various intra-cluster spacings, we found that an intra-cluster spacing of 600 ft. produced the most clusters greater than size 7 in both the September 25 and the July 11 data file. The September 25 data set yielded 652 clusters, and the July 11 data set yielded 986 clusters using this intra-cluster spacing. Figure 16 shows the impact of varying communication ranges on these sets of clusters.



**FIGURE 16 Impact of communication range for clusters greater than 7 vehicles.**

Figure 16a indicates the dramatic reduction in the percentage of vehicles in each data set that experience a collision as the communication range is increased. When data is received at a range of only 100 ft., an incident produces a collision for nearly 100% of the vehicles. This situation is comparable to an unequipped driver in the fog, where visibility limits the driver's sight to short distances. This result also indicates that very little or no benefit is obtained by utilizing data from ranges greater than 600 ft. or 700 ft. Figures 16b, and 16c show similar results in terms of the average impact velocity of the vehicles involved in collisions, and the average deceleration rate required by the vehicles not involved in collisions, respectively.

As in the experiment from the previous section, we also performed the same experiment utilizing the same intra-cluster spacing, but including all the clusters, and not just those larger than 7 vehicles. Including all clusters greater than 2 vehicles, the September 25 data set yielded 1755 clusters, and the July 11 data set yielded 2043 clusters. Figure 17 shows the impact of varying communication ranges on these sets of clusters.

The results of Figure 17 again indicate that there is significant gain in terms of reduction in the number and severity of incidents, and the braking force required to avert a collision for communication ranges up to 600 ft. or 700 ft., but that very little or no benefit is obtained for communication ranges above 600 ft. or 700 ft.

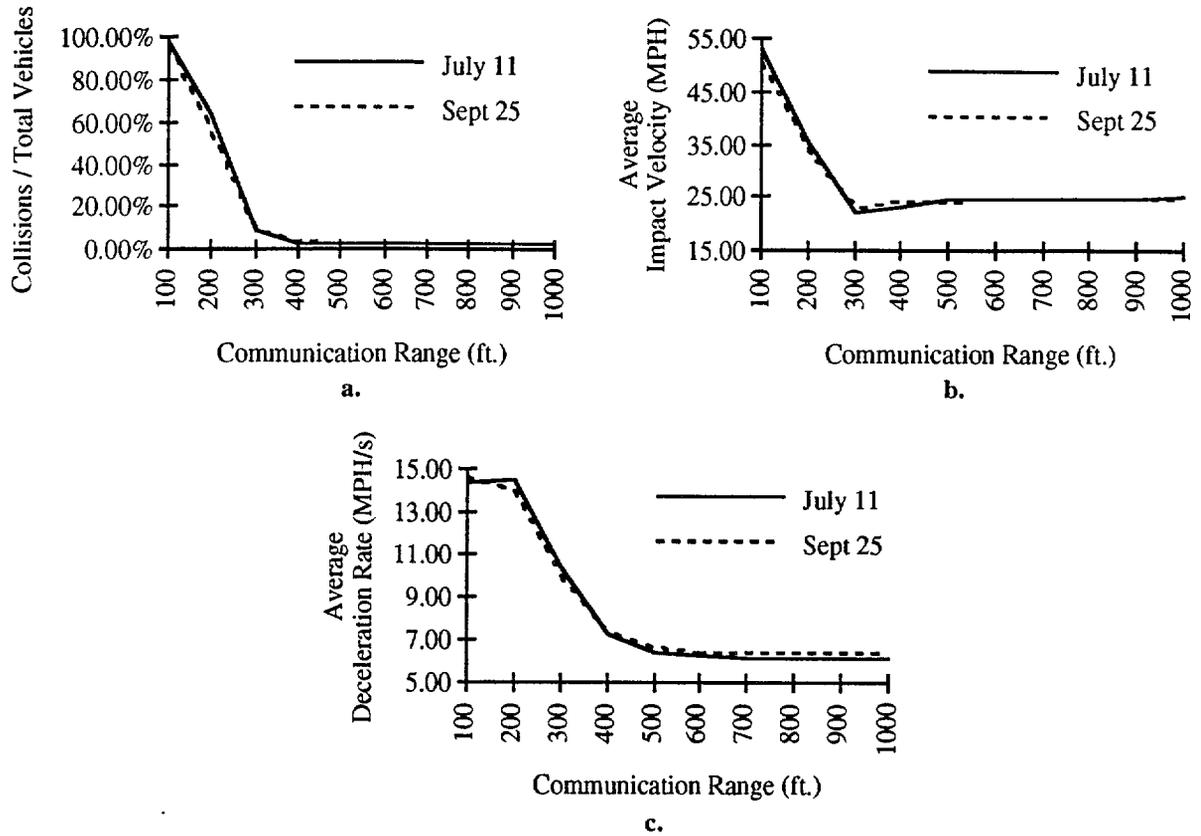
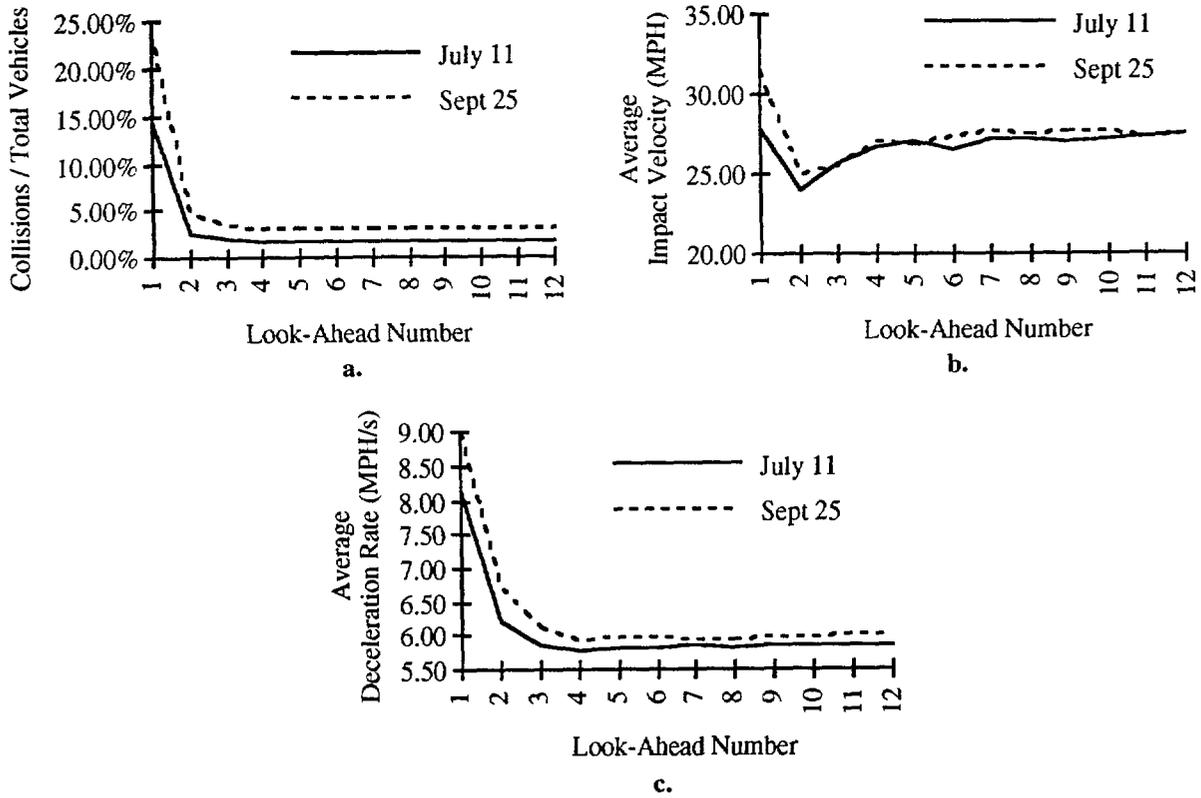


FIGURE 17 Impact of communication range for clusters greater than 2 vehicles.

### 3.1.3 Severity of Lead Vehicle Braking Action

Both of the above experiments utilized a random lead vehicle deceleration uniformly distributed between 30% and 100% of the vehicle's maximum deceleration. This section gives the results of performing both of the above experiments with the lead vehicle deceleration chosen as the vehicle's maximum deceleration. This is the worst case scenario where the lead vehicle's driver fully applies the brake.

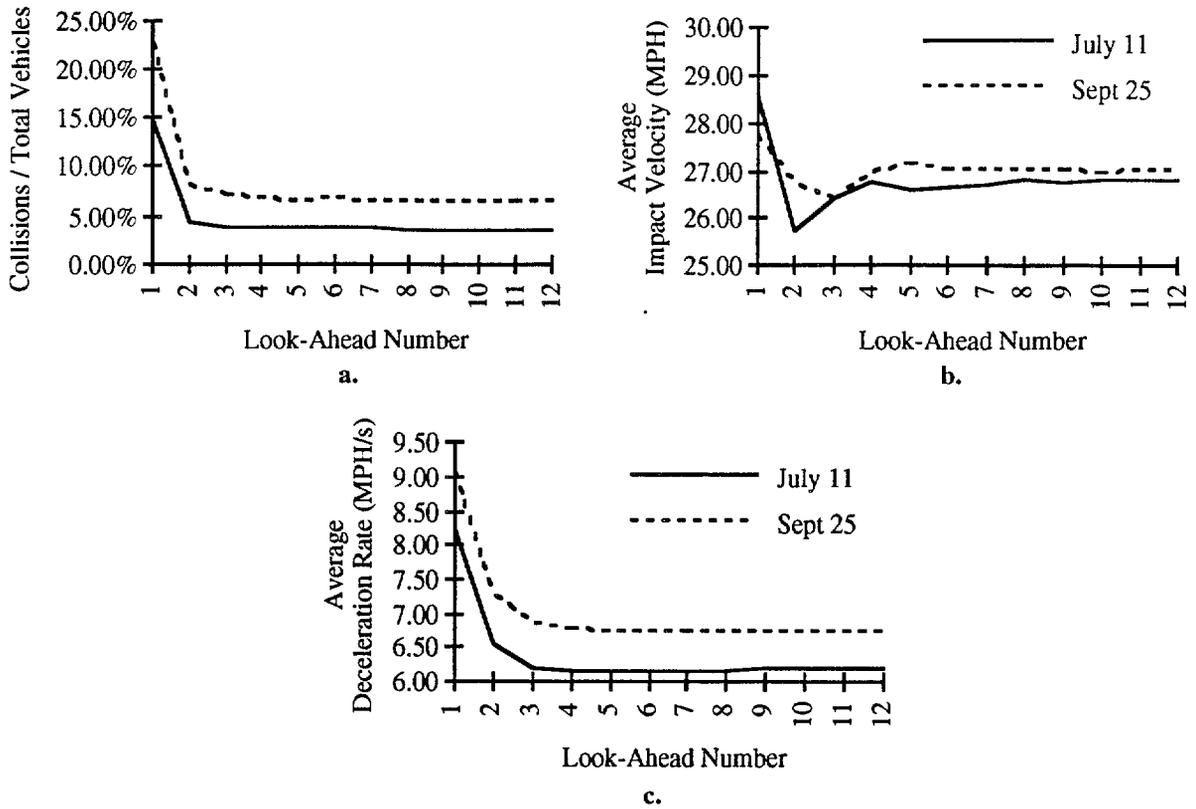
Figure 18 shows the impact of varying look-ahead numbers on the same sets of clusters utilized to produce the results shown in Figure 14.



**FIGURE 18 Impact of look-ahead number for clusters greater than 12 vehicles.**

Comparing Figures 14 and 18, we see a higher percentage of collisions, a higher impact velocity, and a higher required deceleration rate in Figure 18, which one would expect with a higher lead vehicle deceleration rate, but the conclusions to be drawn are the same. The results again indicate that looking ahead more than 6 or 7 vehicles produces little or no benefit in the system.

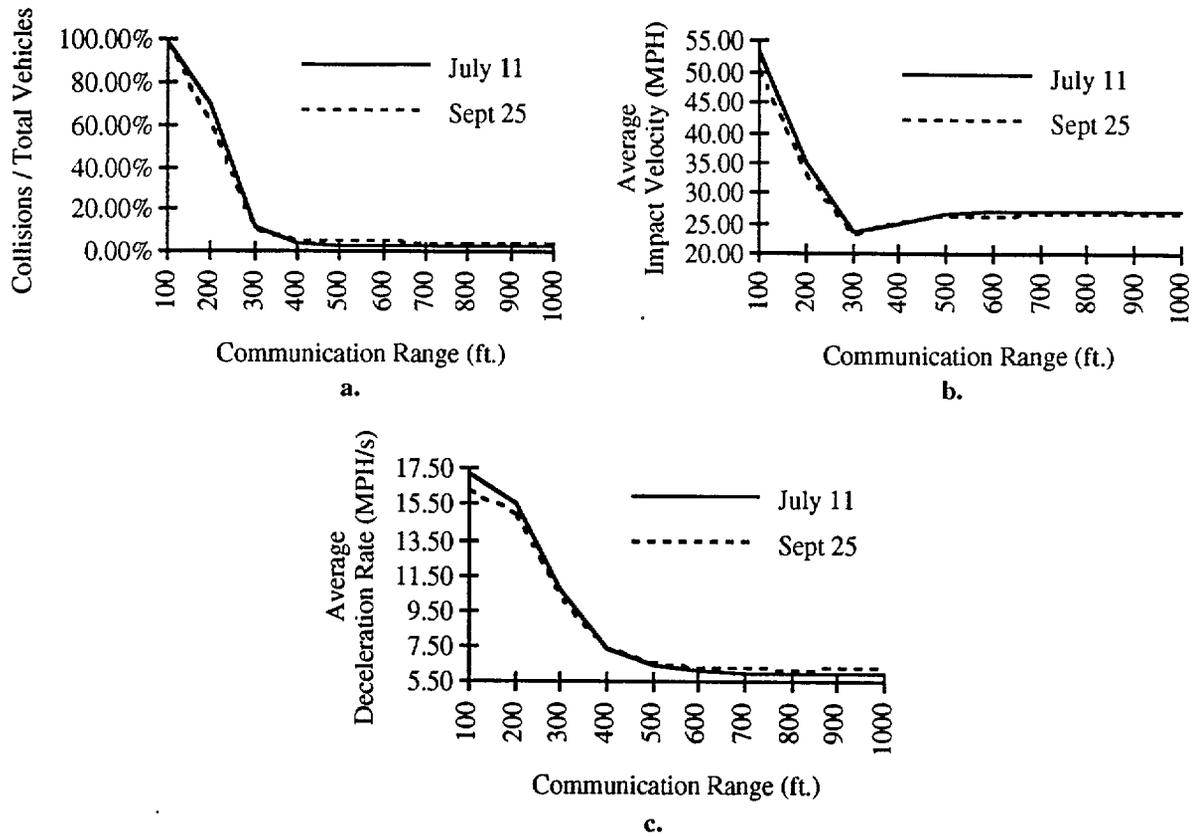
Again, we also performed the same experiment including all the clusters, and not just those larger than 12 vehicles. Figure 19 shows the impact of varying look-ahead numbers on these sets of clusters.



**FIGURE 19 Impact of look-ahead number for clusters greater than 2 vehicles.**

If we compare the results of Figure 19 to its counterpart, Figure 15, we again see a higher percentage of collisions, a higher impact velocity, and a higher required deceleration rate in Figure 19, but the conclusions are still the same.

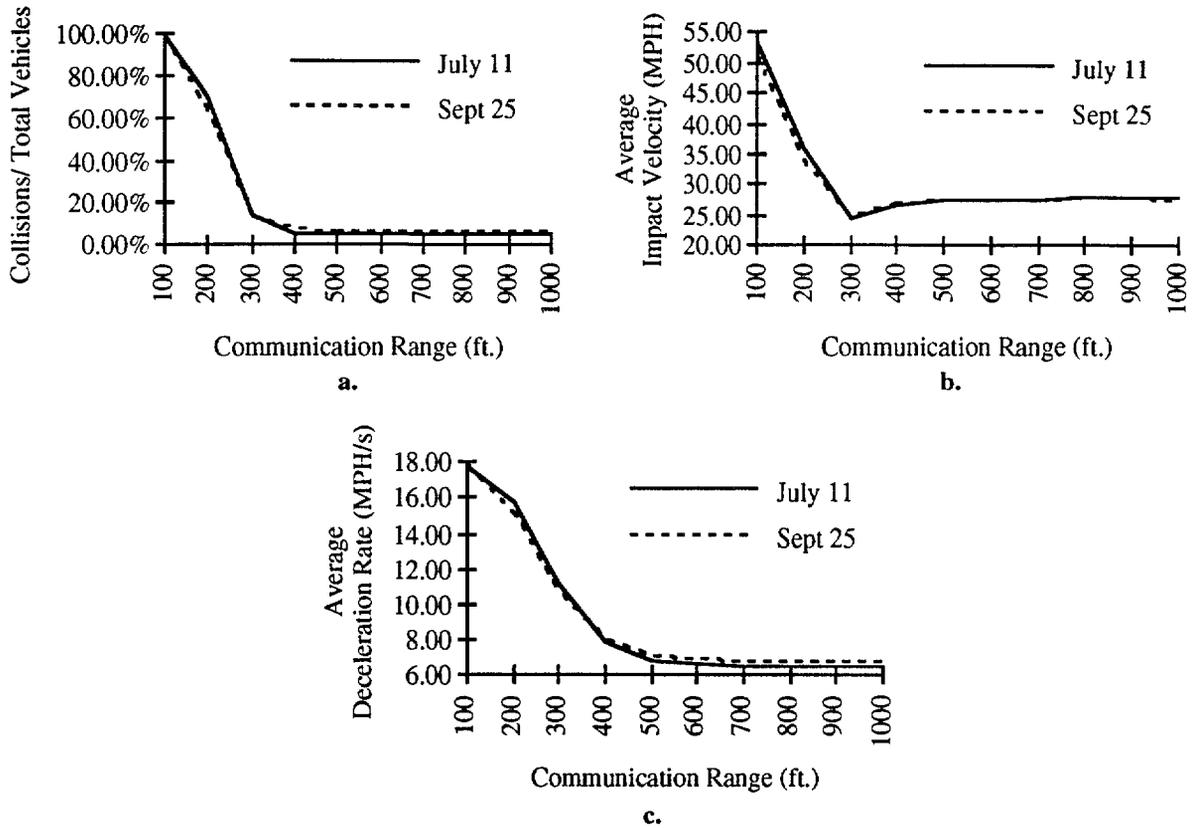
Fixing the look-ahead number at 7 vehicles, we performed the experiment of Section 3.1.2 with maximum lead vehicle deceleration. Figure 20 shows the impact of varying communication ranges on the same sets of clusters utilized to produce the results shown in Figure 16.



**FIGURE 20 Impact of communication range for clusters greater than 7 vehicles.**

The conclusions to be drawn from the results shown in Figure 20 are the same as those drawn from Figure 16. Very little or no benefit is obtained by utilizing data from communication ranges greater than 600 ft. or 700 ft.

Again, we also performed the same experiment including all the clusters, and not just those larger than 7 vehicles. Figure 21 shows the impact of varying communication ranges on these sets of clusters.



**FIGURE 21 Impact of communication range for clusters greater than 2 vehicles.**

The results of Figure 21 indicate, as in the previous section, that there is significant gain in terms of reduction in the number and severity of incidents, and the braking force required to avert a collision for communication ranges up to 600 ft. or 700 ft., but that very little or no benefit is obtained for communication ranges above 600 ft. or 700 ft.

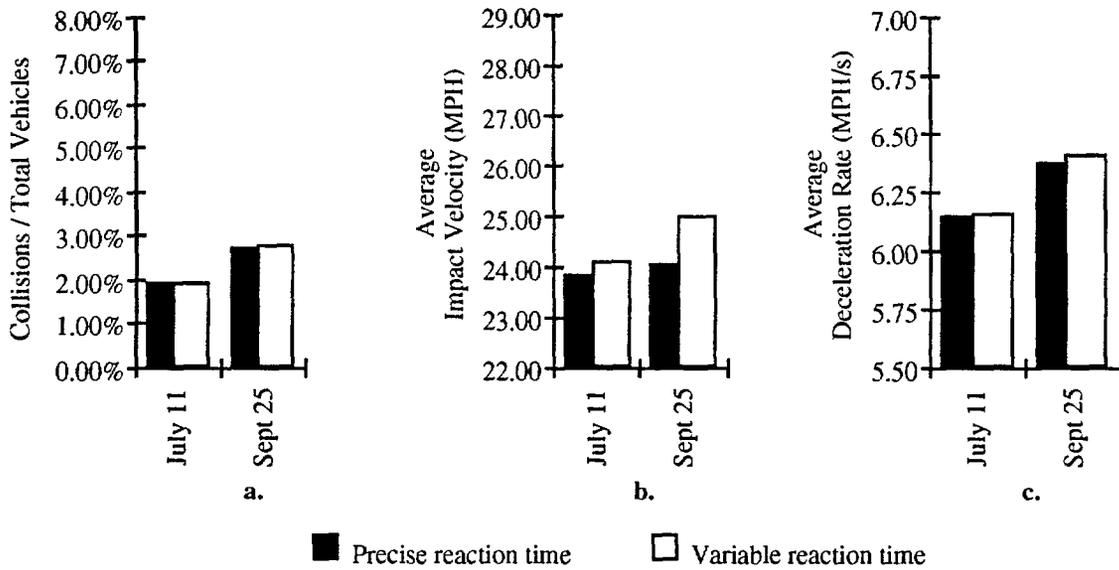
The results of this section indicate that look-ahead number and communication range are unaffected by the severity of the lead vehicle's deceleration.

**3.1.4 Variation in Driver Reaction Times from the Expected Value**

For this experiment, we fixed the look-ahead number to 7 vehicles, and the communication range to 700 ft., as the results of Sections 3.1.1, 3.1.2, and 3.1.3 suggest. We also selected an intra-cluster spacing of 600 ft., and utilized all clusters greater than 2 vehicles. We assume accurate driver response to the warning system, and the default expected behavior for the lead vehicle.

For the first part of this experiment, we chose a lead vehicle deceleration between 30% of the maximum deceleration, and the maximum deceleration possible for the lead vehicle. Since we are studying the effects of variations in driver reaction time, we choose a random reaction time for each driver from the usual lognormal distribution given in Figure 13. We use this value as the system estimate of the driver's actual reaction time. Using this value, we then generate another reaction time to serve as the driver's actual response for this event. This "actual" reaction time is drawn from a normal distribution with a mean value equal to the driver's expected reaction time, and a standard deviation of 0.125 s. Using these values, approximately 95% of the actual reaction times are within 0.25 s. of the expected value. The choice of this normal distribution is somewhat arbitrary. It seems that very little research exists on a particular driver's reaction time repeatability, so we chose this distribution simply to demonstrate the sensitivity of the system to variations in driver reaction time. Further research will need to be done to determine the characteristics of this distribution.

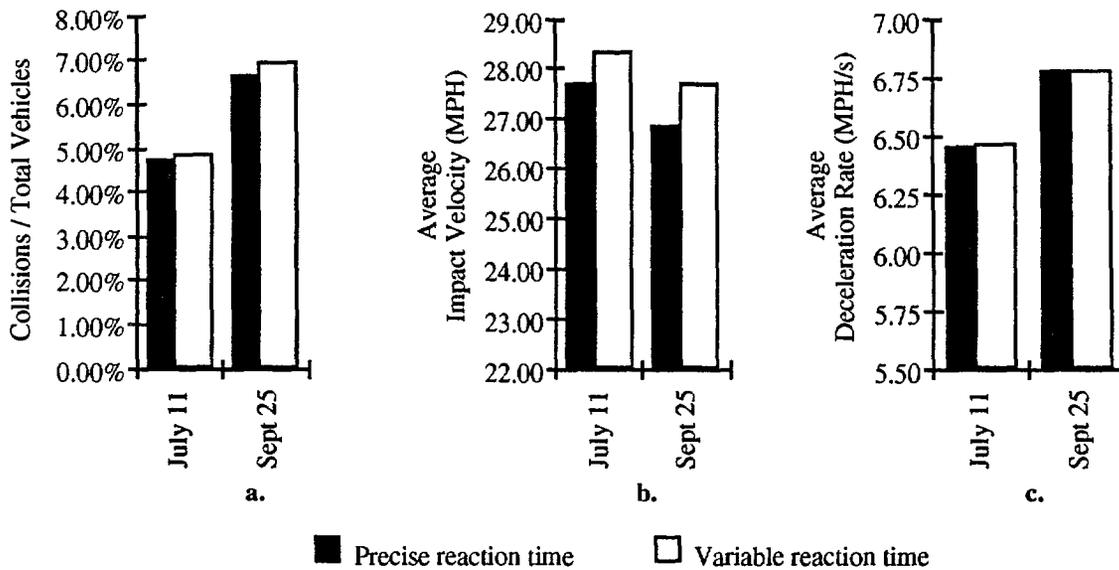
Using this setup, we ran experiments on all the clusters first with the driver's expected reaction time, and the actual reaction time the same, and then again with the actual reaction time variable. Figure 22 shows the effect of variations in the reaction time on the number and severity of collisions, and the braking force required to avert collisions.



**FIGURE 22 Impact of variability in driver reaction time for moderate lead vehicle deceleration.**

The results, in Figure 22a, show that variations in driver reaction time from the value expected by the system cause a slight increase in the percentage of collisions. This occurs because the system's warning is based on its expected driver reaction time. If the driver actually reacts later than expected, a collision may occur. This also produces modest increases in the driver's average impact velocity, Figure 22b, and the driver's average required deceleration rate, Figure 22c. It is important to note that variation in driver reaction time from the expected value produces only minimal increases in the variables of interest.

For the second part of this experiment, we ran the same experiment, but with a lead vehicle deceleration equal to the maximum deceleration possible for the lead vehicle. Figure 23 shows the effect of variations in the reaction time for this case.



**FIGURE 23 Impact of variability in driver reaction time for worst case lead vehicle deceleration.**

Comparing the results in Figure 22 to Figure 23, we again see increases in the percentage of collisions, the impact velocity of the collisions, and the deceleration rate required to avoid collisions, with the increased lead vehicle deceleration rate present in the Figure 23 results, as one would expect. However, the results of Figure 23 are similar to the results of Figure 22 in that the three variables increase only a small amount with variations in driver reaction time.

The results of this experiment show that variability in driver reaction time produces only minimal increases in collisions, impact velocity, and the driver's required deceleration to avoid collisions. The extent of this is dependent on the amount of variability in the reaction time. We have chosen a distribution to demonstrate this sensitivity, but further research will be needed to determine the appropriate distribution for driver reaction time variability, and the corresponding impact on the system.

### 3.1.5 Variation in Degree of Response to the Warning Device

For this experiment, we again fixed the look-ahead number to 7 vehicles, and the communication range to 700 ft., as the results of Sections 3.1.1, 3.1.2, and 3.1.3 suggest. We also selected an intra-cluster spacing of 600 ft., and utilized all clusters greater than 2 vehicles. We assume no variation in driver reaction time from the expected value, and the default expected behavior for the lead vehicle.

Here we are studying the effects of variations in the degree of driver response to the warning device. The assumption is that upon seeing a particular light on the warning device, the driver will produce an associated braking response. Since there are a limited number of lights, in this case 5, there is a range of required decelerations which cause a particular warning to be generated. For example, in the case where the warning device is activated when the deceleration required is between 30% and 100% of the vehicle's maximum deceleration, one light would correspond to a required deceleration between 30% and 44% of the vehicle's maximum deceleration, two lights between 44% and 58%, etc., as illustrated in Figure 24. To simulate driver variability, we assume that upon seeing a particular number of lights the driver responds by producing a deceleration which is normally distributed with a mean equal to the average of the upper and lower percentage of deceleration which produced the light. For example, the center of the range which produces two lights is 51% of the vehicle's maximum deceleration. Thus, the driver would produce a deceleration with a mean of 51% of the maximum deceleration. The standard deviation is chosen such that approximately 95% of the decelerations generated fall in the correct range. For two lights, 95% of the time the deceleration is between 44% and 58% of the maximum. As was the case in the previous section, with a particular driver's reaction time repeatability, it is unclear how a driver will "map" a particular number of lights to a response, so we chose this distribution simply to demonstrate the sensitivity of the system to variations in driver response to the warning device. Further research will need to be done to determine the characteristics of this distribution.

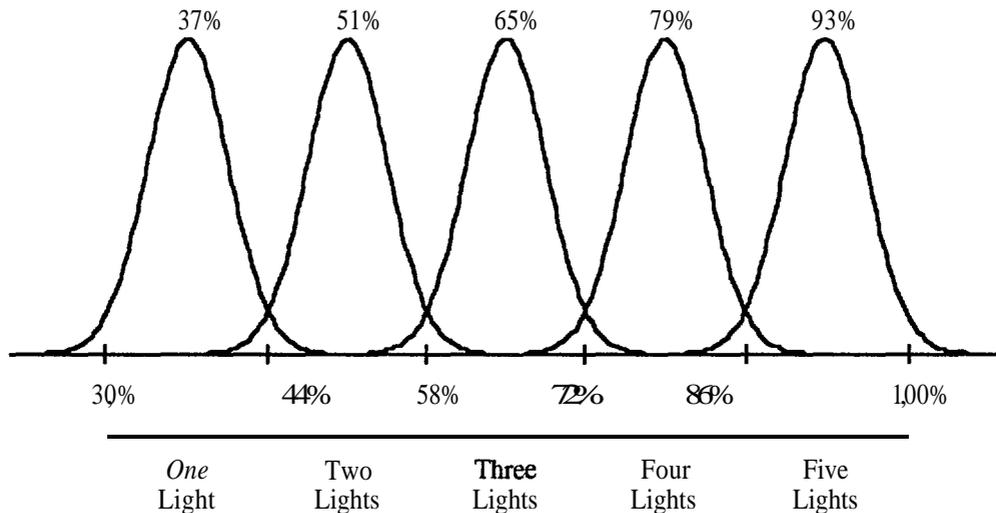
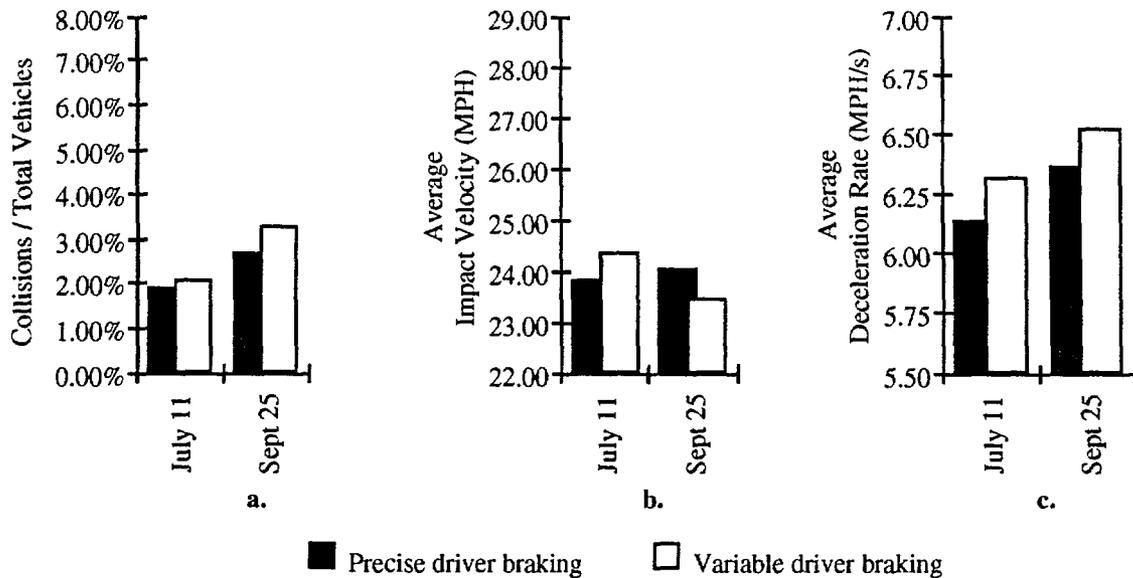


FIGURE 24 Distribution of driver response to a particular number of warning lights.

In the simulation, when the driver reacts to a particular warning level, a response is chosen from the above distribution. This response may be greater than or less than the required response calculated by the algorithm. If the driver brakes harder than required, the warning will eventually decrease. For example, if the initial warning required a deceleration of 60% of maximum, corresponding to three lights, and the driver responded with 67% of maximum, the

required deceleration will drop as time goes on (all other factors remaining the same). Once it drops below 58%, the warning device will change to two lights, and a new response will be chosen centered at 51% of maximum. On the other hand, if the driver produces less deceleration than required, the warning will eventually increase in intensity in a similar fashion.

For the first part of this experiment, we chose a lead vehicle deceleration between 30% of the maximum deceleration, and the maximum deceleration possible for the lead vehicle. Using this setup, we ran experiments on all the clusters first with the driver's deceleration response exactly equal to the required response calculated by the system, and then again with the driver's deceleration response varying according to the above distribution. Figure 25 shows the effect of variations in the driver's deceleration response on the number and severity of collisions, and the braking force required to avert collisions.



**FIGURE 25** Impact of variability in driver braking for moderate lead vehicle deceleration.

Similar to the results of the previous section, the results in Figure 25 show that variations in driver deceleration response from the required value cause a modest increase in the percentage of collisions, as well as the deceleration rate required to avert collisions. The impact velocity of collisions which do occur are reduced slightly for the September 25 data set in Figure 25b. In that case, variations in driver response to the warning device cause additional low impact collisions, which reduce the average impact velocity.

For the second part of this experiment, we ran the same experiment, but with a lead vehicle deceleration equal to the maximum deceleration possible for the lead vehicle. Figure 26 shows the effect of variations in driver response to the warning device for this case.

Comparing the results in Figure 25a to Figure 26a, we see that the percentage of collisions increases from between approximately 2% and 4% in Figure 25a to between 4% and 8% in Figure 26a. This increase is due to the increased lead vehicle deceleration rate. A similar increase can be seen in the average impact velocity and the average deceleration rate, again caused by the increased lead vehicle deceleration rate. If we look at the effect of variations in driver response to the warning device in Figure 26, we see, as was the case in Figure 25, that the variation produces a slight increase in the percentage of collisions, as well as the deceleration rate required to avert collisions.

The results of this experiment show that variability in driver response produces minimal increases in collisions, and the driver's required deceleration to avoid collisions. The extent of this is dependent on the amount of variability in the driver's response. We have chosen a distribution to demonstrate this sensitivity, but further research will be needed to determine the appropriate distribution for driver response to the warning device, and the corresponding impact on the system.

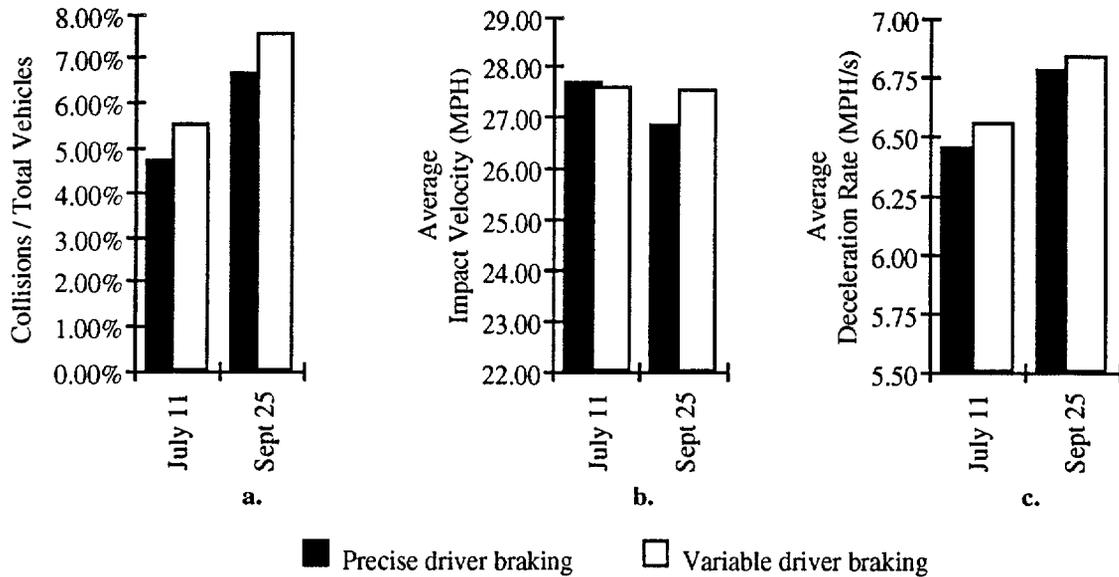


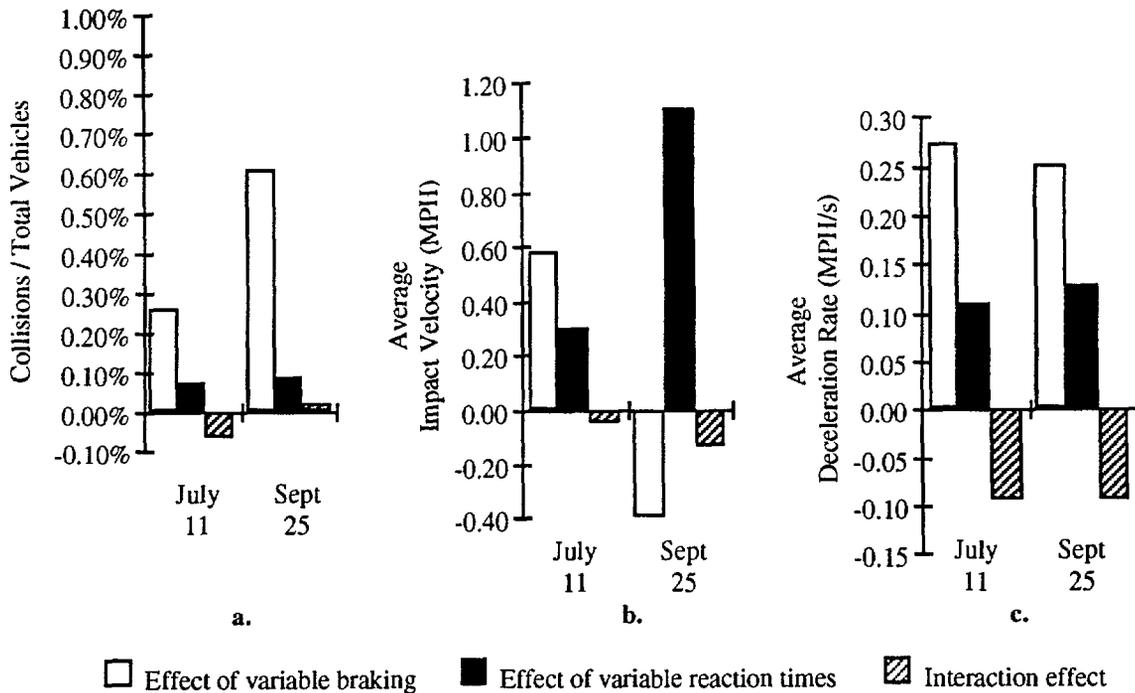
FIGURE 26 Impact of variability in driver braking for worst case lead vehicle deceleration.

**3.1.6 Main and Interaction Effects for Variations in Degree of Response to the Warning Device and Variations in Driver Reaction Times from the Expected**

In Section 3.1.4, we assumed accurate driver response to the warning device, and examined the effects of variations in driver reaction time. In Section 3.1.5, we assumed no variation in driver reaction times from the expected value, and examined the effects of variations in the degree of driver response to the warning device. In this section, we combine these two studies by examining the effect of each of those variations with and without the influence of the other. We will examine the "main effect" of each variation, and the "interaction effect" of their combination. The main effect is the average gain or loss achieved by varying one of the parameters while leaving the other parameters fixed in each of its possible states. For example, to find the main effect of a variation in the degree of driver response to the warning device on the percentage of collisions, we assume no variation in driver reaction times, and find the increase or decrease in the percentage of collisions with accurate driver response to the warning device, and with variable response to the warning device. We then assume variable driver reaction times, and find the increase or decrease in the percentage of collisions with accurate driver response to the warning device, and with variable response to the warning device. The average of these two is the main effect of a variation in the degree of driver response to the warning device on the percentage-of collisions. The interaction effect is, by convention, one half the difference in the average effect of one parameter with or without variation in the other parameter (23).

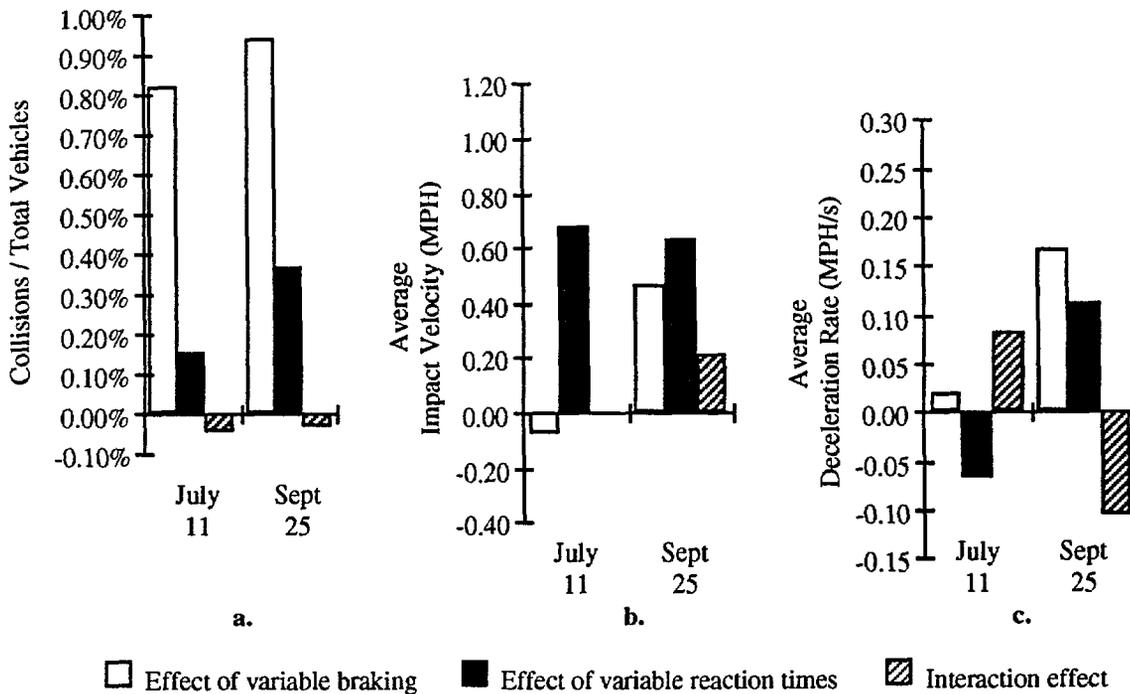
The parameters whose effects produce the largest gain or loss are the most significant parameters. Figure 27 shows the effects of variation in response to the warning device, variation in driver reaction times, and the interaction of the two for lead vehicle deceleration between 30% and 100% of maximum.

Figure 27a and 27c indicate that the effect of variations in driver response to the warning device is more significant than the effect of variations in driver reaction time, and is more significant than the interaction of the two. It is important to note here that these variations depend largely on the assumptions made about the variations in driver response to the warning device, and the assumptions made about the variations in driver reaction times.



**FIGURE 27** Effects of variability in driver braking and reaction time for moderate lead vehicle deceleration.

Figure 28 shows the same effects for a worst case lead vehicle deceleration. The results again indicate that the effect of variations in driver response to the warning device is the more significant effect.



**FIGURE 28** Effects of variability in driver braking and reaction time for worst case lead vehicle deceleration.

### 3.1.7 Assumptions about the Behavior of the Furthest Vehicle Ahead

As discussed in Section 2.2.7, the algorithm must make an assumption about the behavior of the vehicle furthest ahead from which data has been received. Since we have no knowledge of any vehicles ahead of this vehicle or their expected action, we need to make an assumption about what the furthest vehicle ahead will do in the next time interval (i.e. the 10 ms). To this point, we have been assuming that this vehicle will continue at its current rate of acceleration, positive or negative, during the next interval. This is the most likely assumption, but it is not the most conservative. The most conservative approach would be to assume worst case behavior for this vehicle. That is, to assume that the furthest vehicle ahead from which we have data will decelerate with its maximum deceleration rate.

In experimenting, we found that this assumption causes severe braking for all of the vehicles in the cluster, which in most cases is unnecessary. Since the algorithm receives data and recalculates the required response often, for example every 10 ms, any changes in the furthest vehicle ahead's deceleration rate will be incorporated into the driver's warning rapidly. Also, since the furthest vehicle ahead from which we have data is, just that, furthest away, its impact is less than nearby vehicles, and thus the impact of any assumption is lessened. For this reason, it seems reasonable to assume that the vehicle will continue at its current rate of acceleration, positive or negative, during the next interval.

## 3.2 SYSTEM DEPLOYMENT

In this section, we fix several of the parameters studied in the previous section, and focus on system penetration, and driver reliance. Based on the results of Section 3.1, we set the number of vehicles ahead from which the system considers data at 7 vehicles, and we fix the system's communication range at 700 ft. In addition, since the severity of the lead vehicle braking action, variations in driver reaction times from the expected value, and variations in the degree of driver response to the warning device produced minimal variation in the number and impact velocity of collisions, or deceleration rate, for these experiments, we choose a lead vehicle deceleration between 30% and 100% of maximum deceleration, and no variation in driver reaction times, or in response to the warning device.

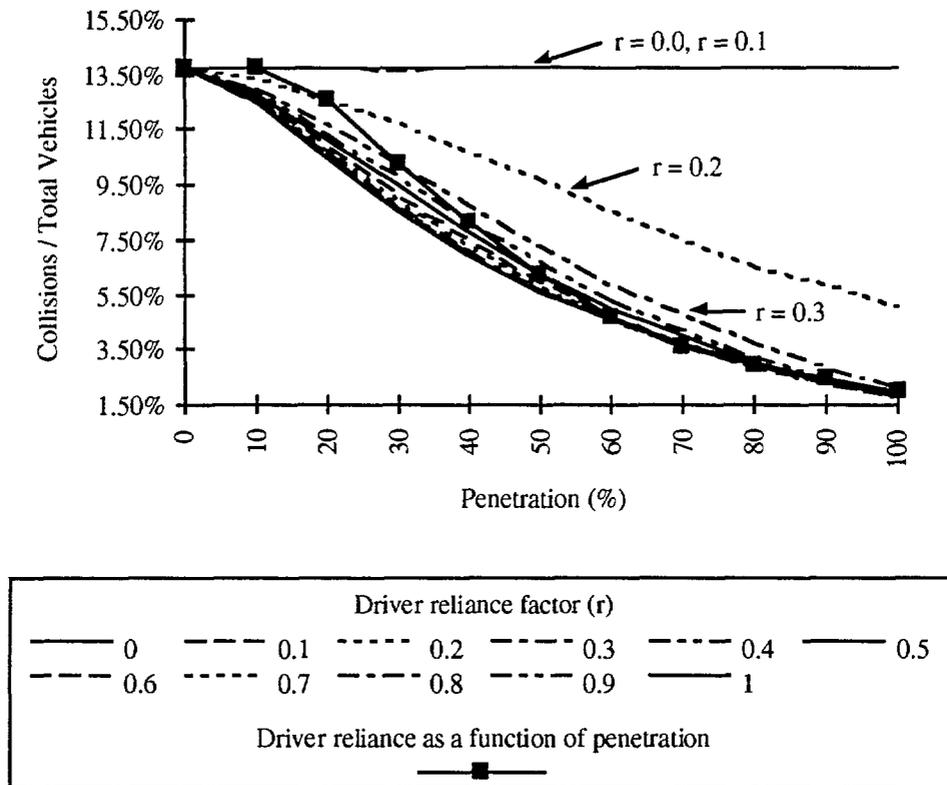
For these experiments, we also have chosen to combine the July 11 and September 25 dam sets, and to focus on the variation in the percentage of collisions. Since the results are similar for each day's data set, and for the other parameters, we have chosen to do this for brevity and simplicity.

For a typical experiment, we choose the number of vehicles ahead which the unequipped driver is capable of seeing (1, 2, or 3). which we'll call *the visual look-ahead*. For each cluster in the combined data set, we choose log-normally distributed random reaction times for each driver, and a normally distributed random deceleration rate for the lead vehicle (see Section 2.1.1). We then vary the penetration level from 0% to 100% and for each penetration level we vary the driver reliance value from 0 to 1. This produces a matrix of results, where each element of the matrix is equivalent to an experiment in Section 3.1. In generating each element of the matrix, we use the same visual look-ahead, the same reaction times, and the same lead vehicle deceleration rate.

Figure 29 shows the results for a visual look-ahead of one. Each line in the figure is a line of constant reliance. In each of the figures in this section, the line which corresponds to  $r = 0$  is a horizontal line. When  $r = 0$ , each equipped driver ignores the system's advice, and thus each driver responds like an unequipped driver regardless of level of penetration. This  $r = 0$  line corresponds to the unequipped driver, values below this line represent improvements over the visual driver, and values above this line correspond to combinations of penetration level and reliance level which produce results which are worse than results with no warning system present.

For each of the lines other than  $r = 0$ , the equipped driver utilizes the advice of the system, to a lesser or greater extent depending on the  $r$  value. Regardless of the  $r$  value though, the results are the same for a penetration level of 0%, since the  $r$  value only affects equipped drivers.

In Figure 29, the  $t = 0.1$  line nearly coincides with the  $r = 0$  line. In this case, with a visual look-ahead of one vehicle, a reliance of 0.1 is essentially the same as not having the warning device at all. A reliance of even 0.2 however, produces significant improvement over the unequipped driver, reducing the percentage of collisions from 13.7% to 5.1% at 100% penetration. Reliance levels of 0.3 and above produce further improvement, reducing the percentage of collisions to 2.0% at 100% penetration. This corresponds to an 85% reduction in the number of collisions when compared to all unequipped drivers ( $r = 0$ ).



**FIGURE 29** Reliance curves for a visual look-ahead of 1 vehicle.

In Figure 29, each of the reliance lines (all the lines but the line with the square markers) is a line of constant reliance. As the system is deployed, from 0% penetration to 100% penetration, it is very unlikely that driver reliance on the system will remain constant. Initially, when the penetration level is low, the driver will most likely place little reliance in the system, but as the penetration of the system increases and the advice provided by the system improves, the driver's reliance will most likely increase. Thus, drivers will not follow a single line of reliance. How the driver's reliance increases with penetration is an area which will require further research, but for the purposes of this paper we will choose a simple possible relationship between penetration and reliance to illustrate a possible deployment trajectory.

In Figure 30, we have plotted three possible relationships between system penetration and reliance. The simplest curve, which has the square markers, depicts a linear relationship between penetration and reliance. One might also speculate that the reliance is low initially, and then increases rapidly above some level of penetration possibly reaching  $r = 1$ , the line with triangle markers, or leveling out at an  $r$  value below  $r = 1$ , the line with the circle markers. For Figure 29, and the other figures to follow in this section, we use the linear relationship.

Though we have chosen a linear relationship between penetration and reliance, the curves of constant reliance in the figures allow the user to plot any other desired relationship by hopping from constant reliance curve to constant reliance curve as the penetration increases, which is how Figure 30 was derived.

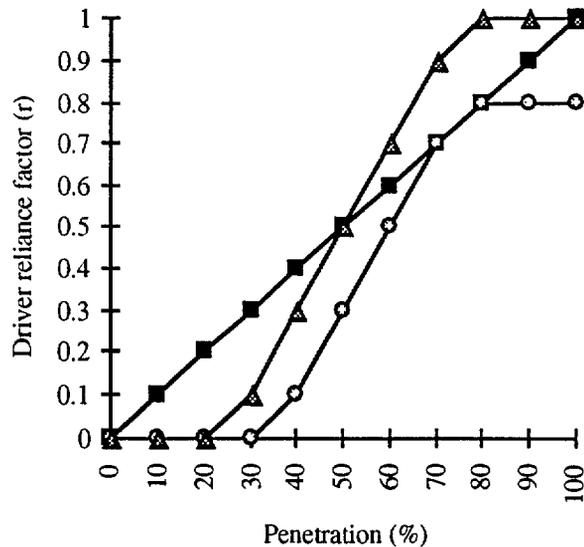


FIGURE 30 Three possible driver reliance vs. system penetration curves.

Looking again at Figure 29, we see the line for driver reliance as a linear function of penetration (the square markers). This curve has a data point on each of the constant reliance curves.

In Figure 29, we used a visual look-ahead of one. This would be the case if the driver could only see the vehicle immediately ahead because that vehicle occluded the view of the next vehicle ahead. This situation is unlikely in reality. We have made the assumption that visual driver look-ahead covers at most three vehicles. In the next two figures, we look at the cases of visual look-ahead equal to two and three, respectively.

Figure 31 shows the results of the same experiment as the results of Figure 29, but with a visual look-ahead of two. This figure is similar to Figure 29, but the  $r = 0.8$ ,  $r = 0.9$ , and the  $r = 1$  each are above the visual driver line ( $r = 0$ ) at low penetration levels. For those curves at low penetration levels, the driver is relying heavily on advice from the warning device which is not very good because of the low level of equipped vehicles.

A point of interest is the penetration level at which the  $r = 1$  line crosses the  $r = 0$  line. At penetration levels below this point, the warning system could produce results which are worse than a fully unequipped set of drivers, assuming perfect visual information for each of two vehicles ahead, and assuming complete reliance ( $r = 1$ ) on the system, which is unlikely at low penetration levels.

For Figure 31, this is the case at penetration levels below approximately 37%, but only for  $r = 0.9$  and  $r = 1$ , and below approximately 10% for  $r = 0.8$ . Thus, if over 37% of the vehicles are equipped, the warning system is an improvement over unequipped drivers for any level of reliance, with a possible reduction in collisions from 2.9% to 2.0%, which is a 31% reduction in the number of collisions.

We again plot (with square markers) the curve for driver reliance as a linear function of penetration. If we follow this curve, we get improvement or remain the same at each level of penetration when compared to all unequipped drivers.

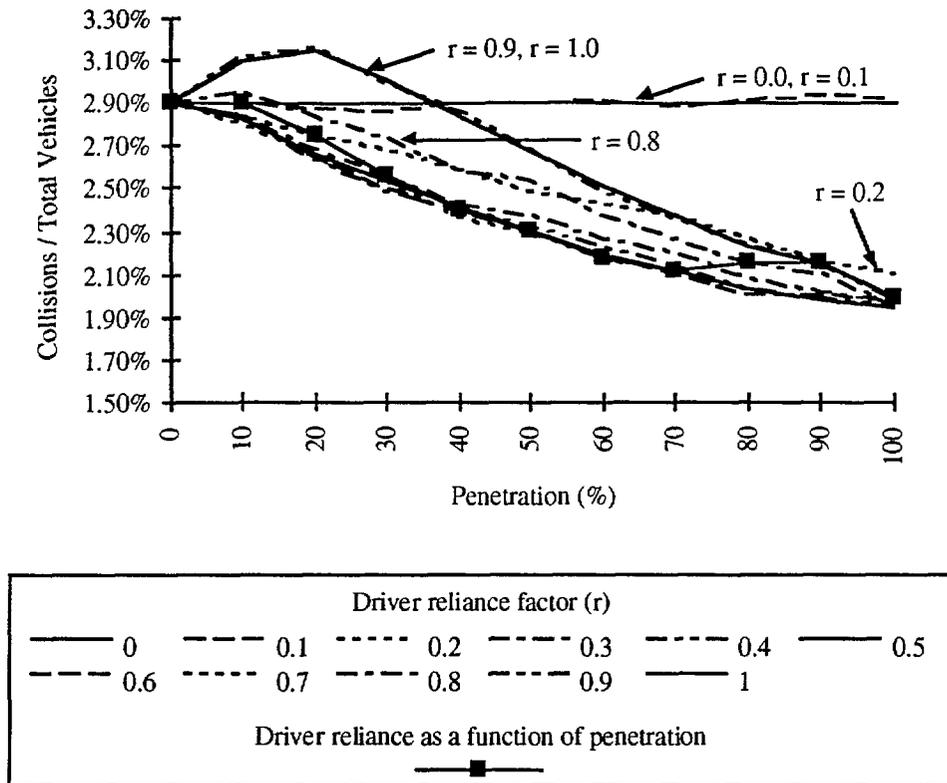


FIGURE 31 Reliance curves for a visual look-ahead of 2 vehicles.

For Figure 32, we ran the same experiment as above, but we used a visual look-ahead of three. This figure is similar to Figure 31, but we have additional reliance lines above the  $r=0$  line at low penetration levels. Also, the penetration level at which the  $r=1$  line crosses the  $r=0$  line is now at approximately 80%, and the possible reduction in collisions is from 2.2% to 2.0%, a 9% reduction in the number of collisions.

Looking at driver reliance as a linear function of penetration (square markers), we get improvement or remain essentially the same as compared to all unequipped drivers at each level of penetration.

It is important to note, that in assuming a visual look-ahead of three, we are using a driver model which is most likely much better than an actual driver. We are assuming perfect determination of position, velocity, and acceleration for three vehicles ahead, and perfect use of that data, when in most cases it is not possible to see the third or even second vehicle ahead. In addition, we are using the same reaction times as the equipped drivers. We limited these reaction times to at most 2.4 seconds, since the warning device would eliminate the long reaction times associated with driver inattention. This is generous for the unequipped driver. Thus, the model we are using for the unequipped driver performs better than an actual driver would be expected to perform. Yet, even using this unequipped driver model we can potentially obtain a 9% reduction in the number of collisions.

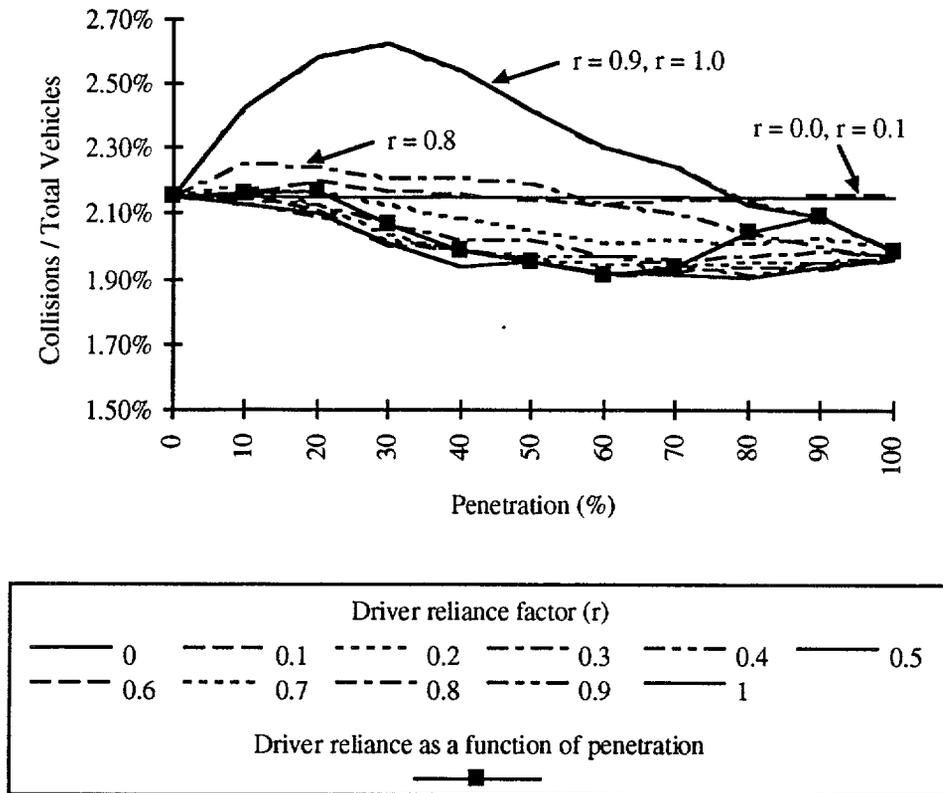


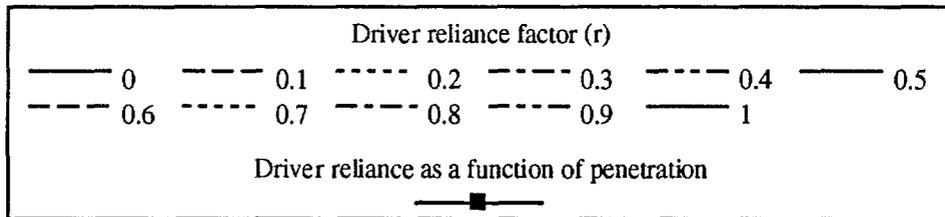
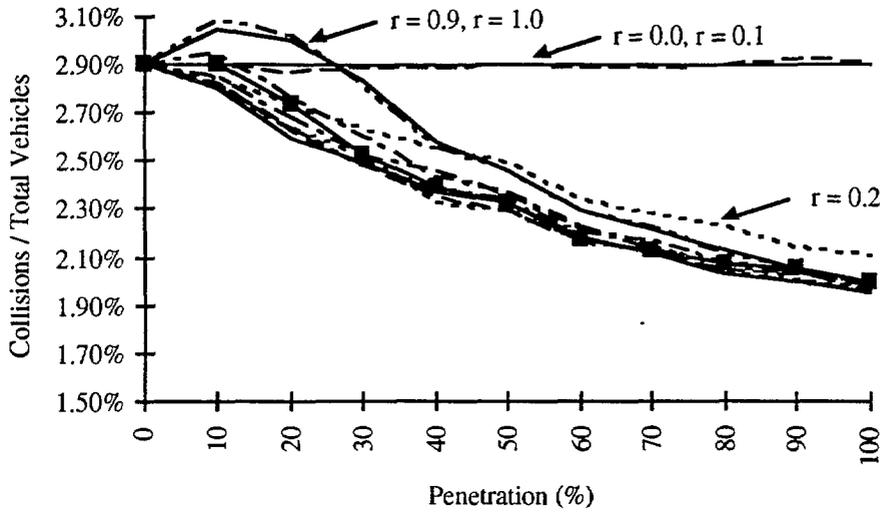
FIGURE 32 Reliance curves for a visual look-ahead of 3 vehicles.

**3.2.1 Broadcast of Data from Non-transmitting Vehicles which are Sensed**

In the experiments presented in Figures 29, 31, and 32, each equipped vehicle in the system transmits data about its own state. Since a vehicle may sense an unequipped (or not transmitting) vehicle ahead, each vehicle could also broadcast the position, velocity, and acceleration of an unequipped vehicle which it senses immediately ahead. We can determine that a vehicle is unequipped since we will sense a vehicle from which no data packet has been received. Broadcasting the data for an unequipped or non-transmitting vehicle increases the communication burden of the equipped vehicle which senses that vehicle, but since the non-transmitting vehicle is not broadcasting, the overall communication burden is not increased as a result.

Figure 33 shows the results of performing the experiment shown in Figure 31\*, with visual look-ahead of two, but allowing equipped vehicles which sense a non-transmitting vehicle immediately ahead to broadcast the state data for the non-transmitting vehicle that the equipped vehicle senses. This produces results similar to those shown in Figure 31 (e.g. 31% reduction in the number of collisions), but the  $r = 1$  line crosses the  $r = 0$  line at approximately 25% as opposed to 37%. Broadcasting data from sensed vehicles increases the effective penetration of the system. For example, if vehicles alternated equipped, unequipped, equipped, unequipped, etc., the effective penetration for equipped vehicles would be 100%. This increase in effective penetration allows for earlier benefits from the system. Also, the peaks of the reliance curves are lowered which improves the worst case behavior of a given reliance curve.

\* We omit the case of a visual look-ahead of one because it is essentially the same as the results of Figure 29.



**FIGURE 33 Reliance curves for a visual look-ahead of 2 vehicles with sensed vehicle data broadcast.**

Figure 34 shows the results of performing the experiment shown in Figure 32, with visual look-ahead of three, but allowing equipped vehicles which sense a non-transmitting vehicle to broadcast the state data for the non-transmitting vehicle that the equipped vehicle senses. This produces results similar to those shown in Figure 32 (e.g. 9% reduction in the number of collisions), but the  $r = 1$  line crosses the  $r = 0$  line at approximately 55% as opposed to 80%. Again, the effective penetration is increased, which provides earlier benefits from the system, and the peaks of the reliance curves are lowered which improves the worst case behavior.

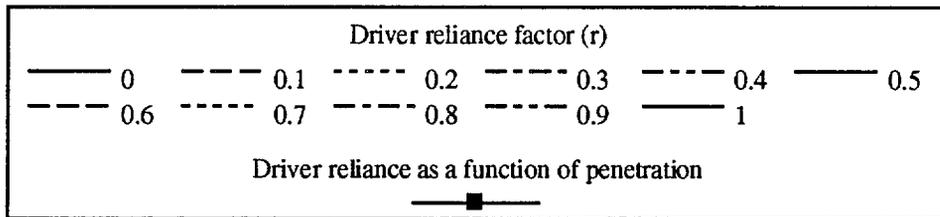
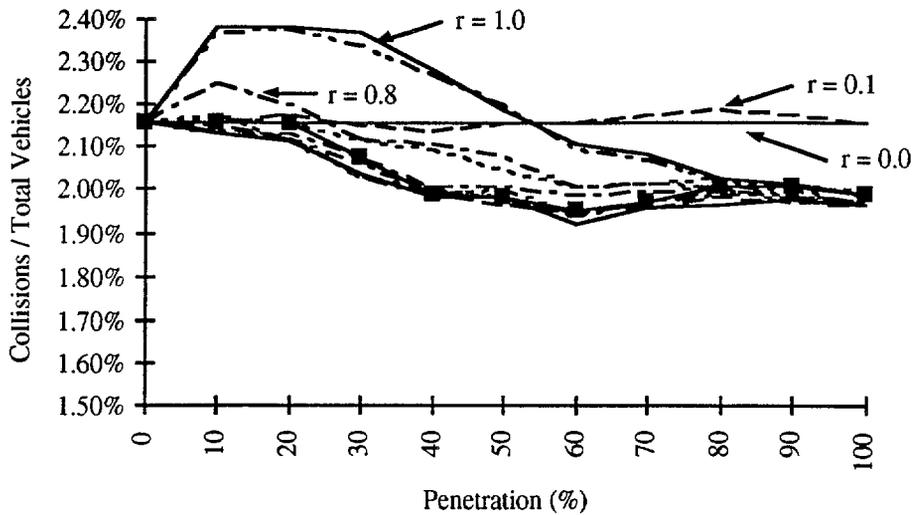


FIGURE 34 Reliance curves for a visual look-ahead of 3 vehicles with sensed vehicle data broadcast.

3.2.2 Limited Visibility (e.g. fog)

For our final simulation experiment, we examine the case of limited sight visibility (e.g. fog). To simulate this we limit the visual driver's sight range to 50 ft. We compare this to the advice provided by the warning device which is assumed not to be limited by the condition causing the limited visibility. We assume, as in the previous experiment, that the system has a look-ahead of 7 vehicles and a communication range of 700 ft.

In Figure 35, the  $r = 0.1$  line coincides with the  $r = 0$  line. In this case, a reliance of 0.1 is essentially the same as not having the warning device at all. A reliance of 0.2 however, produces improvement over the unequipped driver, reducing the percentage of collisions from 99.7% to 86.2% at 100% penetration. A reliance of 0.3 reduces the percentage of collisions from 99.7% to 11.5% at 100% penetration. Reliance levels of 0.4 and above, eliminate all but approximately 2.0% of the collisions.

In the case of limited sight visibility, the system provides dramatic reduction in the number of collisions, even at low levels of driver reliance.

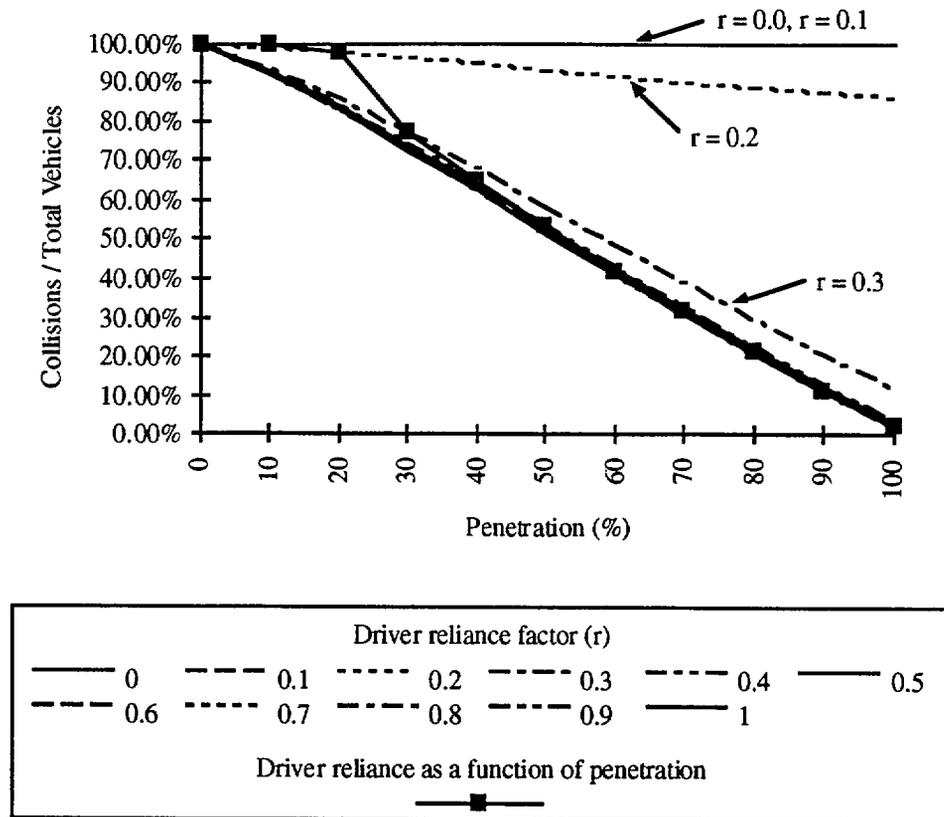


FIGURE 35 Reliance curves for limited visibility.

## 4.0 CONCLUSIONS

We have developed a recursive algorithm which yields the deceleration required by a vehicle,  $V_i$ , to avoid a rear-end collision with the vehicle ahead. This algorithm takes into consideration the response of the vehicle immediately ahead to the vehicle ahead of it, the latter's response to the vehicle ahead of it, and so on, within  $V_i$ 's communication range. The algorithm utilizes more information, and thus provides a more accurate evaluation of the required deceleration than previous distance-warning systems.

To determine the potential benefit of this algorithm, we ran a series of experiments using a quasi-Monte Carlo approach, and data collected by the FHWA. The data used was comprised of two sets of data, each representing a day of traffic data collected on Interstate-40 in New Mexico. To study the impact of the system, we generated an incident by assuming a more or less severe deceleration for a given vehicle in a chain of vehicles. We studied the degree to which the system is effective in 1) reducing the number of collisions, 2) reducing the impact of collisions which do occur, and 3) reducing the required braking force when a collision is averted. We chose these three factors for the following reasons. First, reducing the number of collisions has obvious benefits in terms of reducing injuries and vehicle damage, as well as reducing congestion and delay which accidents produce. Second, reducing the impact of collisions which do occur also translates to reductions in injuries and vehicle damage due to the reduction in the force of the collision. Finally, reducing the braking force when a collision is averted adds to driver comfort, and it also helps to reduce the amount of disturbance introduced in the vehicles following the decelerating vehicle.

Assuming that all the vehicles in the system are equipped (i.e. 100% penetration of the system), we can summarize the conclusions drawn from these experiments as follows:

- significant gain is obtained by processing data from up to 6 or 7 vehicles ahead, but very little or no benefit is obtained by utilizing data from more than 6 or 7 vehicles ahead.
- significant gain is obtained by processing data from vehicles within a 600 ft. or 700 ft. communication range, but very little or no benefit is obtained for communication ranges above 600 ft. or 700 ft.
- the number of vehicles ahead from which to process data, and the appropriate communication range are relatively unaffected by the severity of the lead vehicle's deceleration.
- variability in the driver's reaction time from the expected value has relatively little effect on the number of

collisions, and the deceleration rate required to avert collisions.

- variability in the driver's deceleration rate from the required rate calculated by the algorithm has relatively little effect on the number of collisions, and the deceleration rate required to avert collisions.

We can summarize the conclusions drawn from the deployment experiments (less than 100% penetration) as follows:

- the system is effective in reducing the number of collisions even during deployment.
- broadcasting data from non-transmitting vehicles which are sensed significantly increases the effective system penetration, providing earlier system benefits.
- the system is dramatically effective in reducing collisions in the case of limited sight visibility (e.g. fog)

The above results allow us to place reasonable limitations on the number of vehicles from which to process data and on the communication range required by the system. In addition, they indicate that driver variability has relatively little effect on the potential benefit of the system. Thus, the results of the evaluation indicate that significant benefits in terms of reduction in the number of collisions, reduction in the impact of collisions which do occur, and reduction in the required braking force when a collision is averted are obtained by a system which is capable of processing data from 6 or 7 vehicles ahead, as opposed to processing data from 3 or fewer vehicles ahead which is more typical of an unassisted driver. These benefits are realized even during deployment of the system.

## 4.1 COMMUNICATIONS ARCHITECTURE REQUIREMENTS

The results of the experiments performed provide a basis for specifying the requirements of a communication architecture to support this type of warning device.

The warning device algorithm developed in Section 2.1 requires that each vehicle continuously broadcast its velocity, acceleration, position, and the driver's time remaining until a braking action can occur. The number of bits of information required for each of these depends on the accuracy desired. For the velocity and acceleration, 8 bits are required for an accuracy of 1% (17). We will assume 10 bits for each value. For position, the latitude and longitude can be specified to within 1 meter using 30 bits for each value. That is a total of 60 bits to specify the position. This is a worst case requirement for the position. In addition, the required number of bits could be reduced by having the position be specified relative to some fixed local coordinate. For the time until braking, 10 bits are required to specify a time up to 10 seconds to 2 decimal places.

Under these worst case assumptions, the minimum set of data comprises 90 bits. If we assume an additional 10 bits of information for other data such as road direction and lane number, which would be useful in eliminating irrelevant data packets, we have a total of 100 bits in the minimal data set. Including the typical overhead for error correction of 50% (24), implies that a single packet of data transmitted by a single vehicle will contain approximately 150 bits. In an attempt to establish an estimate of the total bandwidth requirements, we will assume that each vehicle broadcasts a packet every 10 ms. At this rate, a vehicle traveling at 65 miles per hour changes absolute position by less than 10 feet between transmissions. This implies a data rate of 1,500 bits per second for each vehicle in the system.

The result of Section 3.1.2 indicates that a communication range of 600 ft. to 700 ft. will be required to provide the most benefit from the system.

Thus, we have the following requirements:

- 150 bits per data packet per vehicle
- communication frequency of 10 Hz.
- communication range of 700 ft.

Therefore, we need to transmit data at 1,500 bps/vehicle, and each vehicle must be able to receive data from all the vehicles within a communication range of 700 ft. Other researchers have suggested data volumes of 150-500 bits per vehicle at a communication frequency of 10 Hz. with a communication range of 750 ft. (9, 25, 26), for use in cooperative-driving systems. Thus, it seems that the communication requirements proposed are reasonable given current technology.

Spread spectrum techniques may be appropriate for this type of communication, largely due to their ability to provide multiple access to the same frequency, for example via code division multiple access (CDMA), and their low interference to other communications systems.

## 4.2 FUTURE WORK

### 4.2.1 Operational Test

In order to prove the usefulness of the proposed warning device, an operational test will be required. Due to issues of safety which are involved in testing a collision warning device, it is impractical to test the device in normal highway operation. It seems most probable that the device would be tested in a test track environment using professional drivers. Even in this environment, it will not be possible to study the systems effectiveness in reducing collisions, and the impact

velocity of collisions since no collisions should occur in the operational test. We can however study any reductions in deceleration rate for the vehicles involved.

Factors of interest in an operational test, as in the simulation, include look-ahead number, communication range, severity of lead vehicle braking, variations in driver reaction time, and variations in driver braking response to the warning device.

Performing the operational test will require access to several vehicles. We know from the simulation that gains are obtained for look-ahead numbers of up to 6 or 7 vehicles, thus the test would ideally utilize this many vehicles. Each vehicle will need to transmit its acceleration, velocity, position, and the driver's time remaining until a braking action can occur. The on-board system must thus have access to the vehicle's acceleration, velocity, and position. The acceleration can be obtained by the addition of an accelerometer to the vehicle. The velocity can be obtained using the vehicle's equipment by accessing the data provided to the speedometer. Since the algorithm requires the relative position of the vehicles, for the operational test, we can provide the distance traveled by each vehicle from each vehicle's known starting point. Thus, the vehicle's position is obtained using dead-reckoning.

Since we have a limited number of vehicles in the operational test, we can greatly simplify the communication system for the test by using a separate channel for each vehicle in the test. Thus, each vehicle must be capable of transmitting 150 bps of data on a single channel, and receiving 150 bps of data on several channels.

Each of the approximately seven vehicles must therefore be equipped in the following way:

- computer
- warning device
- accelerometer
- access to speedometer
- dead-reckoning device
- ability to transmit on one channel (1500 bps)
- ability to receive on several channels (1500 bps/channel)
- a sensor which provides the position, velocity, and acceleration of the vehicle immediately ahead

For a typical test, the vehicles will be arranged in a single lane with their relative starting positions known to the on-board computer so that their relative positions can be computed at all times from the distance traveled. To begin a test, the vehicles will be driven on the track until appropriate velocities and headways are obtained. The lead vehicle will then decelerate with a specified severity, and the deceleration rates of the other vehicles will be recorded as the vehicles come to a stop.

Using this type of test, we can study the effects of variations in look-ahead number, communication range, and severity of lead vehicle deceleration as in the simulation. We can also compare the deceleration rates with the system operating to those with the system off for the same test scenario. In addition, we can use these tests to collect data on the variability of a particular driver's reaction time from event to event, and how a driver maps a given warning level to a deceleration response.

It is our intent to use the results of an operational test to adjust the parameters used in the simulation, and to use the simulation results to define the operational test in a cyclic manner.

#### 4.2.2 Lane-Change Advice Issues

The system developed uses data exchanged between vehicles to provide collision warning advisories, but the data exchanged might also be used to provide the driver with lane-change advice. For example, a driver could be advised to change lanes to avoid a slower moving vehicle ahead before braking is required. Since each vehicle in the system would be providing similar advice, the driver-advice algorithm would need to consider the possible actions and reactions of the other vehicles in the system. This is further complicated by the fact that each vehicle will have a slightly different perspective on the local environment, depending on which other vehicles are in its communication range.

Other researchers have addressed a similar problem in the area of air-traffic control (27, 28). When developing an architecture for distributed problem solving (or planning), there are three different approaches: 1) individual planning, 2) aggregate (cluster) planning, and 3) global planning. In the case of the latter two, planning is done by one or more regional processors. Such regional or global processing introduces problems of reliability and scalability, since numerous vehicles rely on a single processor. Further, regional or global processing would add significant amounts of infrastructure to the system. For these reasons, it is unlikely that such processing would be utilized, but it should not be ruled out as a possible option. If we restrict ourselves to an individual or distributed planning approach, where each vehicle processes incoming data and provides advice locally, we must consider the two general classes of individual planning: autonomous and cooperative (27). In an autonomous approach, each vehicle's processor decides on an action to take without communicating its decision to the other vehicles and without knowing what actions the other vehicles in the system have decided to take. In the cooperative approach, the vehicles exchange information regarding their decisions and may alter their decisions based on this exchange. This type of communication and plan-altering may proceed for several iterations, resulting in a form of negotiation.

It seems unlikely that extensive communication or negotiation will be possible, however, due to the limited communication bandwidth and the need for timely decision making. Nevertheless, some information in addition to the velocity, acceleration, and position could be exchanged to assist the decision process of the vehicles. This information could comprise the current recommended deceleration and lane selection, and possibly driver desires such as an intended route or planned lane change recommended by a route guidance device, or the speed set on a cruise-control device. This data could be used to encourage a form of cooperation among vehicles. For example, a driver could be advised to accelerate for the purposes of providing a “gap” for a vehicle changing lanes or entering the roadway.

In an individual planning system, each vehicle receives data from a limited number of vehicles in its local area. For the purposes of collision warning, it appears that data from the vehicles within several hundred feet is sufficient to provide valuable advice to the driver. In the case of lane-change advice, the appropriate communication range would need to be determined. It is unclear whether valid lane-change advice could be provided without extending the communication range, and thus the bandwidth requirements.

Even if the appropriate data could be exchanged, a major issue involves determining the performance metric used to evaluate alternative advice scenarios. Each vehicle in the system would continuously consider the state of the vehicles around it, and possibly provide the driver with a lane-change recommendation. In providing the recommendation, the system would consider the possible lane-change advice provided to the other vehicles by those vehicles’ on-board systems. In a given situation, there may be numerous combinations of lane-selection options for the vehicles involved which could reduce overall delay. For example, in a given situation, it might be advisable for a vehicle to stay in its current lane, and the slower-moving vehicle ahead to move to the right. On the other hand, in the same situation, the vehicle could move to the left, allowing the slower-moving vehicle to stay in its lane. The best alternative would depend on the state of the other vehicles in the system. With numerous vehicles being considered, several alternatives arise. Each alternative must be evaluated against the other alternatives to determine the best option. To do this, each alternative must have a net benefit (for example, reduction of total delay) associated with it, which can then be compared to the net benefit of the other options. We must also consider here whether the value of the net benefit be based on system optimality or individual optimality. That is, is the best option the one that is best for all of the vehicles considered, or the one that is best for the individual vehicle performing the evaluation? Establishing the appropriate performance metric so that the possible options can be compared is an important issue in developing a lane-change advice system.

The issue of determining the appropriate performance metric is further complicated by the fact that each vehicle in the system will have a slightly different perspective on the local environment due to its limited communication range. Vehicles that are on the edge of each other’s communication range will possibly have very different views of the local environment because they each have in common communication with only the vehicles between them. The vehicles behind the trailing vehicle and ahead of the lead vehicle are outside the other’s communication range. On the other hand, vehicles that are in close proximity to one another will have quite similar views of the local environment. This may provide some help in developing the system since the vehicles that are closest to one another, and thus whose actions have the greatest impact on one another, have similar views of the local environment, whereas the vehicles that are farther apart and thus have possibly different views of the local environment have less influence on each other. Though this effect diminishes the impact of differing perspectives, the issue of differing views of the environment will need to be addressed.

Finally, in developing a lane-change advice system, it is important that the system be non-manipulable. That is, that an individual driver cannot be allowed to manipulate the system to his or her advantage by providing false input to the system (e.g., a false desired speed), or by strategically disregarding the advice provided by the system. Such system abuse could lead to a lack of trust in the advice provided by the system, and thus widespread non-use of the system. If in fact it seems possible for an individual driver to manipulate the system, some type of incentive system (e.g., a monetary incentive) might be necessary to discourage this type of behavior.

## APPENDIX

We wish to solve for the required deceleration  $a_1(b_1)$ , and the time,  $e_1$ , when the conditions  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$  are satisfied. The equations for  $v_1(t)$ ,  $s_1(t)$ ,  $v_2(t)$ , and  $s_2(t)$  are given in Section 2.1.2.2 as Equations 5, 6, 1, and 2 respectively. We designate these equations here as Equations A.1, A.2, A.3, and A.4 respectively. Thus, we have

$$v_1(t) = v_1(b_1) + a_1(b_1) \cdot (t - b_1) \quad (\text{A.1})$$

$$s_1(t) = s_1(b_1) + v_1(b_1) \cdot (t - b_1) + \frac{1}{2} \cdot a_1(b_1) \cdot (t - b_1)^2 \quad (\text{A.2})$$

where,

$$v_1(b_1) = v_1(0) + a_1(0) \cdot b_1, \text{ and}$$

$$s_1(b_1) = s_1(0) + v_1(0) \cdot b_1 + \frac{1}{2} \cdot a_1(0) \cdot b_1^2,$$

and

$$v_2(t) = v_2(\tau_2) + a_2(\tau_2) \cdot (t - \tau_2) \quad (\text{A.3})$$

$$s_2(t) = s_2(\tau_2) + v_2(\tau_2) \cdot (t - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (t - \tau_2)^2 \quad (\text{A.4})$$

where,

$$\tau_2 = \begin{cases} 0, & t \leq b_2 \\ b_2, & b_2 < t \leq e_2 \\ e_2, & t > e_2 \end{cases}$$

To simplify the solution, we let  $T = t - b_1$ . This implies  $t - \tau_2 = T + (b_1 - \tau_2)$  and  $(t - \tau_2)^2 = T^2 + 2 \cdot (b_1 - \tau_2) \cdot T + (b_1 - \tau_2)^2$ . We can therefore rewrite Equations A.1, A.2, A.3, and A.4, producing Equations A.5, A.6, A.7, and A.8 respectively.

$$v_1(t) = v_1(b_1) + a_1(b_1) \cdot T \quad (\text{A.5})$$

$$s_1(t) = s_1(b_1) + v_1(b_1) \cdot T + \frac{1}{2} \cdot a_1(b_1) \cdot T^2 \quad (\text{A.6})$$

$$v_2(t) = v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) + a_2(\tau_2) \cdot T \quad (\text{A.7})$$

$$s_2(t) = s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 + v_2(\tau_2) \cdot T + a_2(\tau_2) \cdot (b_1 - \tau_2) \cdot T + \frac{1}{2} \cdot a_2(\tau_2) \cdot T^2 \quad (\text{A.8})$$

We want  $s_2(t) - s_1(t) = L_2$ . Substituting Equations A.8 and A.6 gives

$$s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 + v_2(\tau_2) \cdot T + a_2(\tau_2) \cdot (b_1 - \tau_2) \cdot T + \frac{1}{2} \cdot a_2(\tau_2) \cdot T^2 - [s_1(b_1) + v_1(b_1) \cdot T + \frac{1}{2} \cdot a_1(b_1) \cdot T^2] = L_2.$$

Combining terms gives Equation A.9.

$$\frac{1}{2} \cdot [a_2(\tau_2) - a_1(b_1)] \cdot T^2 + [v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)] \cdot T + [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2] = 0 \quad (\text{A.9})$$

In order to eliminate  $T$  from Equation A.9, we use the requirement that  $v_1(t) = v_2(t)$ . Substituting Equations A.5 and A.7 into  $v_1(t) = v_2(t)$  gives

$$v_1(b_1) + a_1(b_1) \cdot T = v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) + a_2(\tau_2) \cdot T,$$

and solving for  $T$  produces Equation A.10.

$$T = \frac{v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)}{[a_1(b_1) - a_2(\tau_2)]} \quad (\text{A.10})$$

We can now eliminate  $T$  in Equation A.9 by substituting the expression for  $T$  from Equation A.10. This produces

$$\begin{aligned} & \frac{1}{2} \cdot [a_2(\tau_2) - a_1(b_1)] \cdot \left[ \frac{v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)}{[a_1(b_1) - a_2(\tau_2)]} \right]^2 \\ & + [v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)] \cdot \left[ \frac{v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)}{[a_1(b_1) - a_2(\tau_2)]} \right] \\ & + [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2] = 0 \end{aligned}$$

Simplifying this gives

$$\begin{aligned} & -\frac{1}{2} \cdot \frac{[v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)]^2}{[a_1(b_1) - a_2(\tau_2)]} + \frac{[v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)]^2}{[a_1(b_1) - a_2(\tau_2)]} \\ & + [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2] = 0 \end{aligned}$$

Combining like terms yields

$$\frac{[v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)]^2}{2 \cdot [a_1(b_1) - a_2(\tau_2)]} + [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2] = 0$$

At this point, we can solve for the required deceleration,  $a_1(b_1)$ , producing Equation A.11.

$$a_1(b_1) = \frac{-[v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)]^2}{2 \cdot [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2]} + a_2(\tau_2) \quad (\text{A.11})$$

Equation A.11 yields the deceleration required by vehicle  $V_1$  when it begins a braking action at time  $b_1$ . This equation is independent of  $t$ , and is based strictly upon the initial positions, velocities, and accelerations of the vehicles involved, and the expected behavior of the lead vehicle. By substituting Equation A.11 into Equation A.10, we can determine the time,  $e_1$ , where the conditions,  $v_1(t) = v_2(t)$  and  $s_2(t) - s_1(t) = L_2$ , are satisfied, and the current braking action can be terminated. Thus we have

$$T = \frac{v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)}{\left[ \frac{-[v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)]^2}{2 \cdot [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2]} + a_2(\tau_2) \right] - a_2(\tau_2)},$$

which simplifies to

$$T = \frac{-2 \cdot [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2]}{v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)}.$$

By definition  $T = t - b_1$ , thus at time  $t = e_1$ , we have  $T = e_1 - b_1$ , or

$$e_1 = \frac{-2 \cdot [s_2(\tau_2) + v_2(\tau_2) \cdot (b_1 - \tau_2) + \frac{1}{2} \cdot a_2(\tau_2) \cdot (b_1 - \tau_2)^2 - s_1(b_1) - L_2]}{v_2(\tau_2) + a_2(\tau_2) \cdot (b_1 - \tau_2) - v_1(b_1)} + b_1. \quad (\text{A.12})$$

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